

1. You are sending information over a channel as a sequence of 0's and 1's. However, due to the technical limitations of the receiver, it is not possible to send two consecutive 1's (doing so could cause them to be perceived as a single 1, leading to errors). How much information can you send over such a channel in a message of length n ?

More precisely, let M_n be the message space consisting of all $\{0, 1\}$ -strings of length n that do not contain consecutive 1's and where all the messages have the same probability. What is the entropy of this message space?

2. At https://en.wikipedia.org/wiki/Letter_frequency, you can find a table giving the frequency of letters in English texts. For simplicity, suppose that an English text is a random sequence of n letters, each chosen independently at random with the probability given by the aforementioned table. What is the entropy of such a text?

3. You are playing the game of "guess a number" with your friend: He picks an integer between 1 and n and you want to determine the number by asking him a series of questions. The questions can only be of form "Is your number greater than k ?" for an integer k . You have played this game with him so often that you know for each $i \in \{1, \dots, n\}$ the probability p_i that he chooses the number i ; let H be the entropy of the corresponding probability space. Show that you can win the game using at most $H + 2$ questions on average.