1. You have a deck of cards with 4 suits and 13 values per suit (52 cards in total). You shuffle it in any way and deal 13 different piles, each pile containing 4 cards. Show that you can always select exactly one card from each pile so that the 13 selected cards have all 13 different values. Hint: Consider the bipartite graph joining each value to all piles that contain a card of this value. Given a subset of values, how many piles must (at least) contain cards whose value belongs to this subset?

2. Let d be a non-negative integer. Show that a graph G has an orientation such that every vertex has outdegree at most d if and only if every subgraph F of G has at most $d|V(F)|$ edges. Hint: Consider matchings in the the bipartite graph where one side is formed by the edges of G, the other side contains d copies of each vertex of G, and each edge uv is adjacent to all copies of u and v.

3. Use the previous exercise to show that every planar graph has an orientation such that every vertex has outdegree at most three.