1. Let G be a graph with at least two vertices, let v be a vertex of G, and let e be an edge of G. Show that

$$\lambda(G) - 1 \leq \lambda(G - e) \leq \lambda(G)$$
$$\kappa(G) - 1 \leq \kappa(G - e) \leq \kappa(G)$$
$$\kappa(G) - 1 \leq \kappa(G - v)$$

   On the other hand, show that

   - there exists a graph G and a vertex $v \in V(G)$ such that $\lambda(G) \geq 10^6$ and $\lambda(G - v) = 0$, and
   - there exists a graph G and a vertex $v \in V(G)$ such that $\lambda(G) = \kappa(G) = 0$ and $\lambda(G - v) \geq 10^6$.

2. Let G be a graph and let k be a positive integer. For vertices $u, v \in V(G)$, we write $u \sim v$ if $u = v$ or G contains at least k pairwise edge-disjoint paths from u to v. Show that the relation $\sim$ is an equivalence. Hint: It will be useful to prove that if $u \sim v \sim w$, then every edge-cut in G separating u from w has size at least k.

3. Let G be a graph of maximum degree at most three. Show that for every positive integer k, if G is k-edge-connected, then G is k-connected. Hint: For contradiction, suppose that G contains a cut of size less than k, and show you can turn it into an edge-cut of size less than k using the assumption that all vertices have degree at most three.