

1.3 Homework

1. For a non-negative integer n , let c_n be the number of strings of length n consisting of the letters **a**, **b**, and **c** that do not contain the substrings **aa**, **ab**, **ba**, and **bb**. Thus, $c_0 = 1$, $c_1 = 3$, $c_2 = 5$, and $c_3 = 11$. Show that

$$c_n = c_{n-1} + 2c_{n-2}$$

holds for every $n \geq 2$.

2. Show that $c_n = \Theta(2^n)$.
3. Show that the following two quantities are equal for every $n \geq 1$:
 - The number of strings of length n consisting of letters **a** and **b** and starting with **a**.
 - The number of ways to express n as a sum of positive integers, where the order of the terms in the sum matters.