Practicals for Introduction to Approximation and Randomized Algorithms

WS2324 - 4. practical

1 Parallel colouring

Come up with a Las Vegas parallel algorithm for graph colouring that uses n processors to colour a graph with n vertices and maximum degree Δ using 2Δ colours in expected time $\mathcal{O}(\Delta \log n)$.

2 k-Center problem

- We are given a metric space with vertices V and metric d.
- For any given subset $S \subset V$ and any vertex $v \in V$ we can compute the distance $d(v, S) = \min_{s \in S} d(v, s)$.
- The "necessary radius" of S is then defined as $R(S) = \max_{v \in V} d(v, S)$.
- We are given $k \in \mathbb{N}$ and are looking for a set S such that $|S| \leq k$ and $R(S)$ is as small as possible.
- Come up with a 2-approximation algorithm.

3 Independence

For this task, consider only random variables with distribution $Bern(0.5)$, a.k.a. uniformly random bits.

- Find k random bits that are $k-1$-independent but not k-independent.
- How many 2-independent random bits can you make from k fully independent random bits?

HW7: k-Supplier problem

For a given $k \in \mathbb{N}$, a set of m "suppliers", a set of n "consumers", and a metric defining the distances between all these points, find a subset of k suppliers such that the maximum distance between a consumer and their nearest supplier is as short as possible.

Come up with a 3-approximation algorithm.

HW8: Better colouring

We are given a graph with maximum degree Δ. Coming up a sequential greedy algorithm for colouring such graphs using at most $\Delta + 1$ colours should be trivial. Come up with a Las Vegas parallel algorithm that can colour such a graph using at most $\Delta + 1$ colours with high probability. Δ can be a parameter of your algorithm - processors know the value of Δ ahead of time. Try not to assume that Δ is a small number.

Information

- There will be ten homework tasks in total, each worth four points.
- You have two weeks to solve it.
- You need at least 25 points to pass.
- Submit homework via Owl.

Link: `https://kam.mff.cuni.cz/owl/c/zs2324/apxr/`
Enroll token: 6de8d9714087