Practicals for Introduction to Approximation and Randomized Algorithms

WS2324 - 2. practical

1 Rolling dice

Imagine that you want to select a uniformly random number from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$ but only have a regular six-sided dice at your disposal. How could you do it? What is the expected number or dice rolls needed? Try and get as close to the optimum as possible.

2 Limits

Prove that Christofides algorithm is not better than a 3/2-approximation. More formally, describe an infinite series of graphs such that the ratio between the solution found by Christofides and the optimum converges to 3/2.

3 Sorting

We have three (miserable) algorithms for checking that an array with n elements is sorted. Each of them repeatedly select a uniformly random pair of elements and check that those are in the correct order. A pair can be selected repeatedly. All algorithms return FALSE whenever they find a pair not in the correct order. The algorithms return TRUE when:

- It has checked all $\binom{n}{2}$ pairs.
- It sees a pair for the second time.
- After checking n pairs. Additionally, it returns FALSE if it sees a pair for the second time.

Each of these algorithms belongs to one of the classes BPP, RP and ZPP. Assign each to its correct class and prove, that it really belongs there. If needed, show how to modify the algorithm to have the required error probability.

HW3: Reduction

Assume only that finding the shortest Hamiltonian path is NP-hard.

Prove that both finding the shortest Hamiltonian path and finding the shortest Hamiltonian cycle are NP-complete problems.

HW4: Algorithms

In the Vertex Cover problem, we are looking for the smallest subset of vertices of a graph such that each edge of the graph has at least one endpoint in this set. Consider the following simple algorithm:

While the graph contains at least one uncovered edge, select an arbitrary uncovered edge and add both of its endpoints into the subset.

Show that this is an approximation algorithm and determine its approximation ratio.

Information

- There will be ten homework tasks in total, each worth four points.
- You have two weeks to solve it.
- You need at least 25 points to pass.
- Submit homework via Owl.

Link: `https://kam.mff.cuni.cz/owl/c/zs2324/apxr/`
Enroll token: 6de8d9714087