Practicals for Introduction to Approximation and Randomized Algorithms

WS2324 - 1. practical

1 Repeated coins

We are given a fair coin (one where you get heads with probability exactly 0.5) and flip it eight times. Determine the probability of the following events (a formula using exponentiation, sums, binomials, factorials, etc. is enough):

1. The number of heads is equal to the number of tails.
2. The number of heads is strictly greater than the number of tails.
3. i-th and $(9-i)$-th flip have the same result for all $i \in \{1, 2, 3, 4\}$.
4. There are at least four heads in a row. (Hint: Try it with fewer flips first.)

2 Biased coin

We have a "biased" coin which ends up on heads with unknown probability p. Use multiple flips of this coin to simulate a single flip of a fair coin. What is the expected number of flips necessary?

3 Fair coin

Now we want to use a fair coin to simulate a biased coin, specifically one which ends up on heads with some given probability p. Determine the number of flips of the fair coin needed in each of the following cases.

1. $p = 1/2^k$ for some $k \in \mathbb{N}$
2. $p = \ell/2^k$ for some $k, \ell \in \mathbb{N}$
3. $p \in \mathbb{Q}$
4. $p \in \mathbb{R}$

Hint: If you get stuck, try reducing the number of flips needed in the previous case.

HW1: Mathematical homework

We have a fair four-sided dice with faces 1, 2, 3 and 4. We roll this dice five times with the results being the random variables A, B, C, D a E.

We define the following four new random variables:

- $W = 2A + B - 1$
- $X = A(A + C)$
- $Y = (A + B + C) \bmod 4$
- $Z = (A + C)(D + E)$

Determine:

1. Which pairs and triples of random variables W, X, Y, Z are independent?
2. What is the expected value of $W + X + Y + Z$?

HW2: Programming homework

What is the probability that, during 50 flips of a fair coin, there will be at least one sequence of at least six heads in a row?

Compute the result with accuracy to at least 0.01 percentage point and then verify the result by simulation. Describe, how did you do that.

Bonus point if the verification is a statistical test of a hypothesis with $\alpha = 0.05$.

Information

- There will be ten homework tasks in total, each worth four points.
- You have two weeks to solve it.
- You need at least 25 points to pass.
- Submit homework via Owl.

Link: `https://kam.mff.cuni.cz/owl/c/zs2324/apxr/`
Enroll token: 6de8d9714087