ARRIVAL: Next stop in CLS

ICALP 2018

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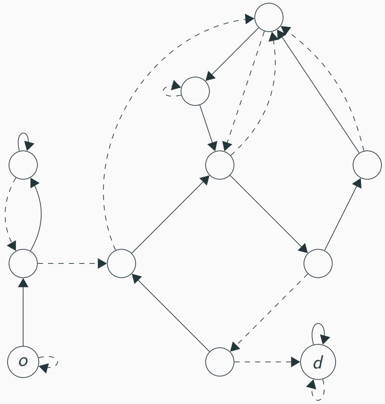
Definition of ARRIVAL

1. Each city... two out-going tracks *even* and *odd*
2. Each visit alternate outgoing edge

Question: If the train starts in origin *o* will it ever arrive to the destination *d*?



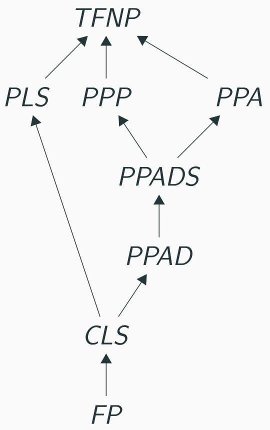
Known results

1. ARRIVAL Dohrau et al.'17:
    - Exponentially long route
    - Solvable in $NP \cap coNP$
2. Karthik C. S.'17:
    - Finding *NP* or *coNP* certificate is in *PLS*
    - Conjecture: search version of ARRIVAL is FPSPACE-complete
3. Fearnley et al.'17:
    - Decision version is *NL*-hard

Our results

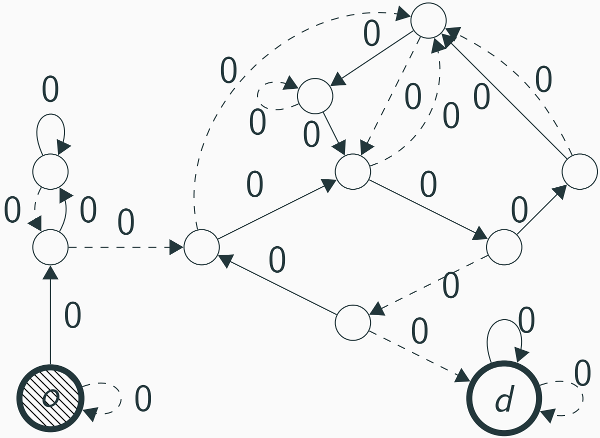
Our results:

1. ARRIVAL is in $UP \cap coUP$
2. Search version in CLS
3. Randomized $\mathcal{O}(1.4143^n)$ algorithm for ARRIVAL



Daskalakis and Papadimitriou'11

Switching flow and Run profile



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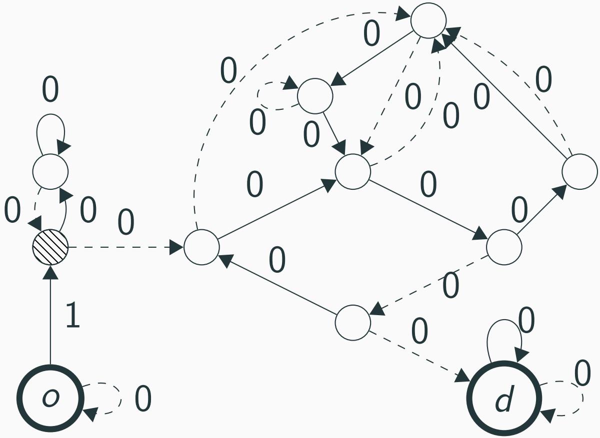

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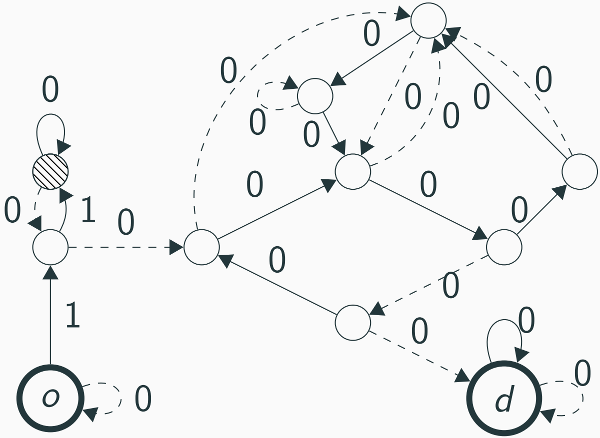

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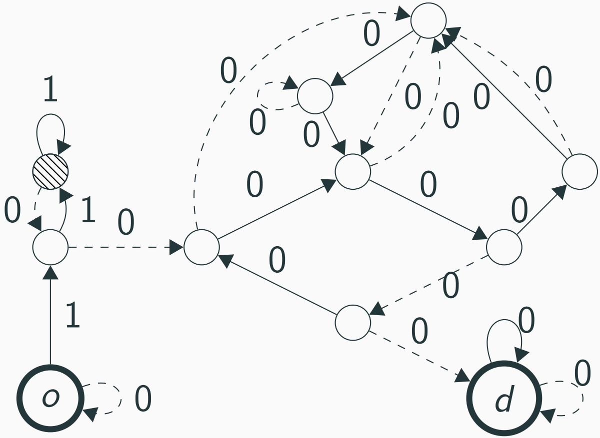

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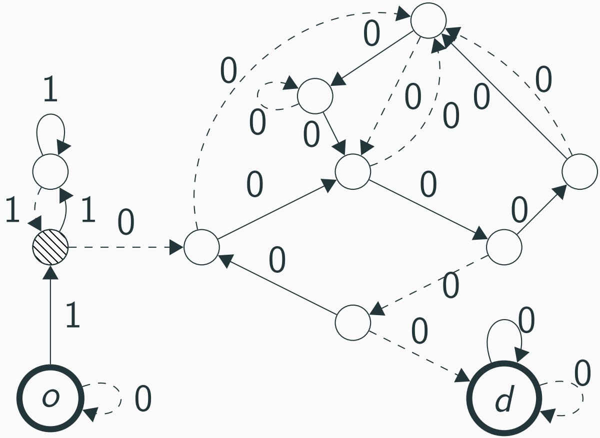

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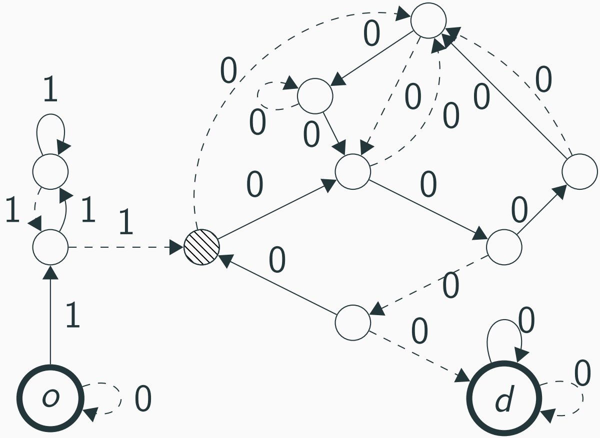
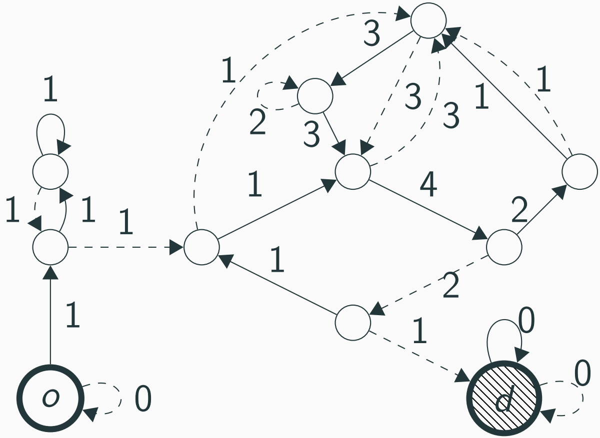


Switching flow and Run profile

Many slides later...

Switching flow and Run profile



Switching flow and Run profile

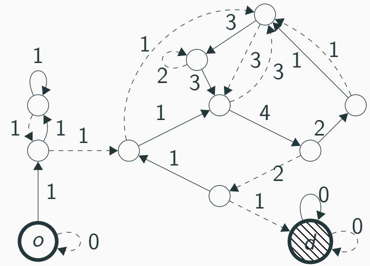
Def: $\boldsymbol{f} \in \mathbb{N}^{2n}$ is a *switching flow* if it satisfies:

1. Kirchhoff's Law (flow conservation)
2. Parity Condition

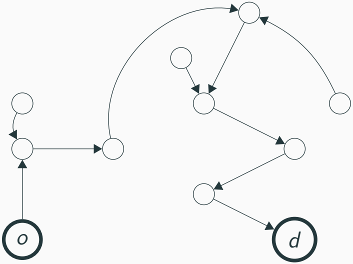
- A *Run profile*. . . route of the train
- (Dohrau et al.'17) *Switching flow* $\Rightarrow$ *run profile*
- *NP* certificate = switching flow
- *coNP* certificate = switching flow to dead-end destination $\overline{d}$

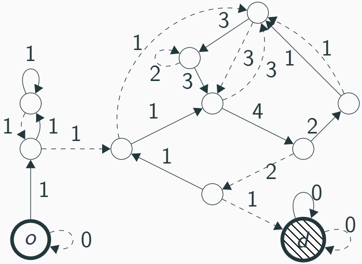
From a switching flow we get:

1. Position of the train
2. Last used edge for each vertex
3. Previous run profile



Graph of last used edges $G_{\boldsymbol{f}}^*$ and $UP \cap coUP$





Run profile verification

Theorem: A switching flow $\boldsymbol{f}$ is a run profile iff the train never left d or $\bar{d}$ and either:

1. There is no cycle in $G^*_{\boldsymbol{f}}$.
2. Or there is exactly one cycle in $G^*_{\boldsymbol{f}}$ which contains the end-vertex of $\boldsymbol{f}$.

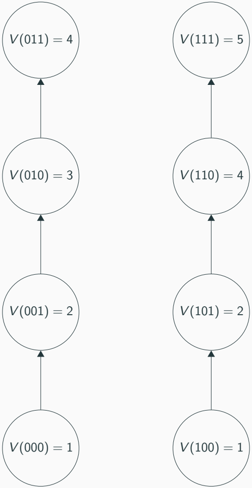
Run profile is unique and we can verify for given vector
$\Rightarrow$ $\textsc{Arrival}$ is in UP.

Continuous Local Search

CLS (Daskalakis and Papadimitriou'11)

- Finding approximate fixed points of contraction maps
- PLCP
- Finding min-max strategy in simple stochastic games
- ...

End of Metered Line



Types of solutions:

1. End of some line
2. Non-trivial beginning of some line
3. Broken valuation:
    - non-trivial $V(x) = 1$
    - or increase by > 1

Theorem (Hubáček and Yogev'17)
EOML is in CLS.

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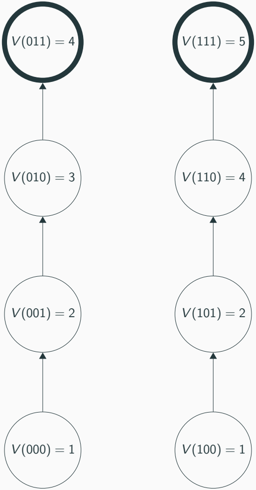

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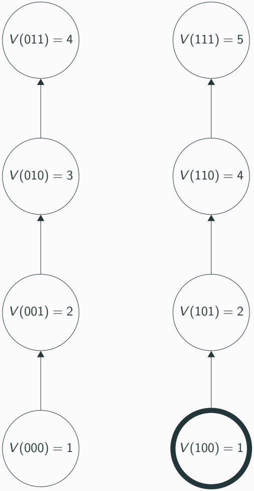

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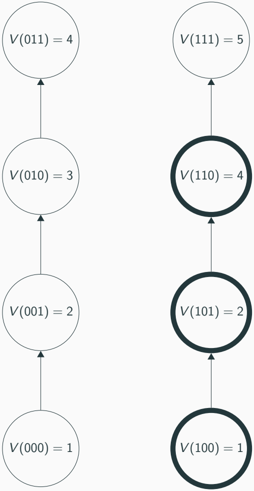

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Reduction ARRIVAL to EOML

Reduction Arrival to EOML:

- Vertices – integer vectors of length $2n$
- If not run profile selfloop and $V(x) = 0$
- Successor/predecessor – next/previous step of the train
- Valuation = sum of the vector

Very special instance of EOML – just one line.

Aldous' Algorithm

EOML algorithm (Aldous'83):

Given EOML instance over strings $\{0,1\}^m$:

1. Let x_m be maximizing $V(x)$ among $2^{m/2}$ random strings.
2. Start from x_m, move to the successor until a solution is found.

Expected number of steps is $\mathcal{O}(m2^{m/2})$.

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The obvious encoding of $\boldsymbol{f}$ (for each edge remember a number that is at most 2^n takes $\Theta(n^2)$ bits) is slower than the trivial simulation.

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Our tool: The switching flow $\boldsymbol{f}$ can be efficiently decoded from the parities of visits of vertices and the position of the train.

$m = n + \log n$ gives us $\mathcal{O}\left(\mathrm{poly}(n)2^{\frac{n+\log n}{2}}\right) \in \mathcal{O}(1.4143^n)$.

Open questions

1. Is ARRIVAL polynomial time solvable?
2. Is there any evidence for not being polynomial time solvable?

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Thank you for your attention!

Formal definitions.

Continuous Local Search

Definition (Daskalakis and Papadimitriou'11)
CLS is the class of total search problems reducible to the following problem called *CLOpt*.

Given two arithmetic circuits $f: [0,1]^3 \to [0,1]^3$ and $p: [0,1]^3 \to [0,1]$, and two real constants $\varepsilon, \lambda > 0$, find either a point $x \in [0,1]^3$ such that $p(f(x)) \leq p(x) + \varepsilon$ or a pair of points $x, x' \in [0,1]^3$ certifying that either p or f is not λ-Lipschitz.

Reminder: Arithmetic circuits are like polynomials but we can reuse intermediate results.

End of Metered Line

Definition (EOML)

Given circuits $S, P\colon \{0,1\}^m \to \{0,1\}^m$, and $V\colon \{0,1\}^m \to [2^m] \cup \{0\}$ such that $P(0^m) = 0^m \neq S(0^m)$ and $V(0^m) = 1$, find a string $x \in \{0,1\}^m$ satisfying one of the following:

1. either $P(S(x)) \neq x$ or $S(P(x)) \neq x \neq 0^m$,
2. $x \neq 0^m$ and $V(x) = 1$,
3. either $V(x) > 0$ and $V(S(x)) - V(x) \neq 1$ or $V(x) > 1$ and $V(x) - V(P(x)) \neq 1$.

The circuits P and S implicitly represent a graph.

The circuit V is a counter of distance from the all zeroes string.

Switching flow and Run profile

We say that $\boldsymbol{f} \in \mathbb{N}^{2n}$ is a *switching flow* if the following holds:

1. Kirchhoff's Law (flow conservation):

$$\forall v \in V\colon \sum_{e=(u,v)\in E} \boldsymbol{f}_e - \sum_{e=(v,w)\in E} \boldsymbol{f}_e = [v = d] - [v = o]\ ,$$

   where $[\cdot]$ is the indicator variable of the event in brackets.

2. Parity Condition:

$$\forall v \in V\colon \boldsymbol{f}_{s_1(v)} \le \boldsymbol{f}_{s_0(v)} \le \boldsymbol{f}_{s_1(v)} + 1\ .$$