1. Compute the inverse matrix using the adjugate matrix: $\begin{pmatrix} 1 & 0 & 1 \\ -2 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$.

2. Compute the determinant of the real matrix: $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & \dots & n \\ -1 & 0 & 2 & 3 & 4 & \dots & n-1 \\ -1 & -2 & 0 & 3 & 4 & \dots & n-1 \\ -1 & -2 & -3 & 0 & 4 & \dots & n-1 \\ -1 & -2 & -3 & -4 & 0 & \dots & n-1 \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \\ -1 & -2 & -3 & -4 & \dots & 1-n & 0 \end{pmatrix}$.

3. Compute determinants of following matrices:

$$\begin{pmatrix} a_1+x & a_2 & a_3 & \dots & a_n \\ a_1 & a_2+x & a_3 & \dots & a_n \\ a_1 & a_2 & a_3+x & \dots & a_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_1 & a_2 & a_3 & \dots & a_n+x \end{pmatrix},$$

$$\begin{pmatrix} x & -1 & 0 & \dots & 0 \\ 0 & x & -1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & x & -1 \\ a_0 & a_1 & \dots & a_{n-1} & a_n \end{pmatrix}, \begin{pmatrix} a+1 & a & 0 & \dots & 0 \\ 1 & a+1 & a & \ddots & \vdots \\ 0 & 1 & a+1 & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & a \\ 0 & \dots & 0 & 1 & a+1 \end{pmatrix}$$

4. Compute determinants of the following matrices:

$$\begin{pmatrix} \sin x & \cos x & 1 \\ \sin y & \cos y & 1 \\ \sin z & \cos z & 1 \end{pmatrix}, \begin{pmatrix} \cos x & \sin x \cos y & \sin x \sin y \\ -\sin x & \cos x \cos y & \cos x \sin y \\ 0 & -\sin y & \cos y \end{pmatrix}, \begin{pmatrix} 1 & \log_b a & \log_c a \\ \log_a b & 1 & \log_c b \\ \log_a c & \log_b c & 1 \end{pmatrix}$$

5. Numbers 697, 476, and 969 are divisible by 17. Without computing the determinant show that the determinant of the following matrix is divisible by 17. $\begin{pmatrix} 6 & 9 & 7 \\ 4 & 7 & 6 \\ 9 & 6 & 9 \end{pmatrix}$

6. Compute the volume of a parallelogram determined by vectors $a^T = (3,1,1)$, $b^T = (2,1,1)$, and $c^T = (2,3,2)$. (A parallelogram in $\mathbb{R}^3$ consists of points which can be written as a linear combination $\alpha a + \beta b + \gamma c$, where $\alpha, \beta, \gamma \in \langle 0, 1 \rangle$.)

7. Let f be a linear map such that $f\colon \mathbb{R}^3 \to \mathbb{R}^3$ maps vectors $a^T = (1,3,1), b^T = (1,0,3), c^T = (1,1,1)$ to vectors $f(a)^T = (3,1,0), f(b)^T = (1,0,2), f(c)^T = (4,1,5)$.

   Determine the volume of the ellipsoid $f(B_3)$ which is the image of a unit ball B_3 (a real ball of unit radius) with respect to the map f.

8. Compute the number of spanning trees in a graph drawn on the blackboard (a tree and a more complicated graph).