NTIN 085 Selected Topics in Comp. Compl. I **winter 2019/2020**

3rd homework assignment - Christmas edition

Turn in by January 7, 2020.

Problem 1. Consider the following algorithm for 3-SAT: *On input ϕ which is a 3-CNF formula on n variables (a conjunction of disjunctive clauses with three literals), pick a random assignment a to the variables and repeat $n/2$-times: if $\phi(a)$ is not satisfied, select a clause which is not satisfied and flip the value of one of its variables chosen uniformly at random. If you ever encounter a satisfying assignment, declare ϕ satisfiable.*

a) What is the success probability of the algorithm in finding a satisfying assignment?

b) Turn this algorithm into a probabilistic algorithm for 3-SAT that runs in time $\mathrm{poly}(n) \cdot 3^{n/2}$ which errs with probability at most $1/n$. Is this algorithm better than a naïve algorithm for 3-SAT?

c) Generalize the algorithm to k-SAT formulas. Is it better than a naïve algorithm for k-SAT?

Problem 2. Consider a random walk $X_0, X_1, X_2, \dots$ on a clique G of size n with self-loops starting from a vertex i. Let j be another vertex, and consider a random walk $Y_0, Y_1, \dots$ of G starting from j. Define a coupled markov chain on pairs of vertices $(X'_0, Y'_0), (X'_1, Y'_1), \dots$ of G that starts from (i, j) so that X'_i is distributed as X_i, Y'_i is distributed as Y_i and the walks couple as fast as possible. What is the expected time for them to couple?

Problem 3. *Modifications of Propp-Wilson:* Consider a markov chain on two states with transition matrix: $\begin{pmatrix} 1/2 & 1/2 \\ 1 & 0 \end{pmatrix}$.

a) What is the stationary distribution of the markov chain?

b) If you start the markov chain from both of the states using the same randomness as in Propp-Wilson algorithm, and you stop once both the chains couple, what is the probability distribution at that moment?

b) For $N_i = 2^i$, imagine that we would run the chain first for N_1 steps then for $N_2 - N_1$ etc., until the chain couples. We output the result at time N_i which is the first time N_i after they coupled. Will we get stationary distribution? Justify your answer.

Problem 4. Consider a two-dimensional grid on $\sqrt{n} \times \sqrt{n}$ vertices with horizontal and vertical edges. Show that the expected cover time is $O(n \log^2 n)$.