NTIN 085 Selected Topics in Comp. Compl. I **winter 2019/2020**

2nd homework assignment - Random walks on graphs

Turn in by December 3rd, 2019.

Problem 1. Consider a path of length n on vertices $1, \ldots, n$. Show that the expected length of a walk that starts at j and reaches either end of the path (vertex 1 or n) is $(n-j)(j-1)$. Show that the expected cover time from j is $E[T_j] = (n-j)(j-1) + (n-1)^2$.

Problem 2. For a d-regular complete rooted tree on n vertices the expected cover time is $\Omega(n \log^2 n / \log d)$. (A d-regular complete rooted tree has all vertices of degree either 1 or d and all leaves are at the same distance from root.)

Problem 3. *Lollipop:* The lollipop on n vertices, where n is even, consists of a complete graph on $n/2$ vertices connected to a path on $n/2$ vertices. Show that the hitting time from any vertex of degree $n/2 - 1$ to the vertex of degree one is $\Theta(n^3)$. *Hint:* Think of a Markov process with $n/2 + 2$ states where all but one vertices of the clique are represented by a single state. Analyze this chain (stationary distribution, hitting times, etc.) to derive the cover time of the lollipop.

Problem 4. *Web:* The web is a graph on $2n$ vertices consisting of a clique on n vertices where each vertex of the clique is connected to a distinct leaf. Show that the cover time of the web is $\Theta(n^2 \log n)$.