NTIN 085 Selected Topics in Comp. Compl. I **winter 2019/2020**

1st homework assignment - Markov processes

Turn in by November 5th, 2019.

Problem 1. Consider an aperiodic, irreducible Markov chain with a transition matrix P. Show that:

a) For all i and j there is some integer M such that for all $t \geq M$, $p_{i,j}^{(t)} > 0$. How large M has to be?

b) There exists M such that for all $t > M$, $\min_{i,j} p_{i,j}^{(t)} > 0$.

c) There exists M such that $\inf_{t \geq M} \min_{i,j} p_{i,j}^{(t)} > 0$.

(The first two items should be proven without using claims about Markov chains. You can use the Chinese remainder theorem.)

Problem 2. Consider a Markov chain $X_0, X_1, X_2, \ldots$ with states $\{1, 2, 3\}$ and a transition matrix P. Find a function $f : \{1, 2, 3\} \rightarrow \{1, 2\}$ and a transition matrix P such that $f(X_0), f(X_1), f(X_2), \ldots$ is not a Markov chain.

Problem 3.

a) Give an example of a Markov chain which has multiple different stationary distributions.

b) Using the fact that an aperiodic, irreducible Markov chain has a unique stationary distribution show that each irreducible Markov chain has also a unique stationary distribution.

Problem 4. Consider the king of chess walking on the chess-board. At each step the king does a random legal move. Define the corresponding Markov chain and find its stationary distribution.