NTIN100 Intro to Info Transmission and Processing summer 2022/2023

3rd homework assignment - Communication complexity

turn in by June 15, 2023.

Problem 1. Let $GT(x, y) : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ be a function that is 1 if and only if $\sum_{i=1}^n x_i \cdot 2^i \geq \sum_{i=1}^n y_i \cdot 2^i$.

a) Show that its deterministic communication complexity satisfies $CC(GT) \leq n+1$.

b) Show that its deterministic communication complexity satisfies $CC(GT) \in \Omega(n)$.

c) Show that its randomized communication complexity satisfies $R_{1/4}(GT) \in O(\log^2 n)$.

d) Show that its randomized communication complexity satisfies $R_{1/4}(GT) \in O(\log n)$.

Problem 2. Let $MED(x, y) : \{0,1\}^n \times \{0,1\}^n \to \{1, \dots, n\}$ be a function that gives the median of the union of the two sets represented by x and y, that is the median of $\{i \in \{1, \dots, n\};\ x_i = 1 \text{ or } y_i = 1\}$.

a) Show a deterministic protocol for MED which communicates at most $O(\log^2 n)$ bits and which works whenever x contains only even integers and y contains only odd integers.

a) Show that its deterministic communication complexity satisfies $CC(MED) \in \Omega(n)$. (*Hint:* Use MED to solve $DISJ$.)

Problem 3. Let $f : X \times Y \to \{0,1\}$ be a function and $M_f \subseteq \{0,1\}^{X \times Y}$ be its matrix. Show that if M_f can be partitioned into ℓ monochromatic combinatorial rectangles then $\text{rank}(M_f) \leq \ell$. Show that $CC(f) \geq \log \text{rank}(M_f)$.

Problem 4. Consider an undirected graph $G = (V, E)$ with m vertices and n edges. Each subset of the edges of G can be represented by a vector $\{0,1\}^n$, where each coordinate corresponds to an edge of G and indicates whether the edge is present in the subset. Define a code $C_{\text{cut}} \subseteq \{0,1\}^n$ of vectors that represent cuts in G, that is subsets of edges $F \subseteq E$ such that for some subset $S \subseteq V$, $F = \{\{u, v\},\ u \in S \ \&\ v \notin S\}$.

a) Show that C_{cut} is a linear code.

b) Show that if we can efficiently find for each $x \in \{0,1\}^n$ the closest codeword from C_{cut}, then we can efficiently find the largest cut in G. Finding the largest cut in G is so called MAX-CUT problem that is known to be NP-complete.