NTIN100 Intro to Info Transmission and Processing summer 2022/2023

2nd homework assignment - Coding and compression

turn in by April 27, 2023.

Problem 1. For each of the following codes decide whether it is uniquely decodable. For each uniquely decodable code find an infinite string which can be decoded in two different ways if there is such a string. Show that prefix-free codes do not have such a sequence.

a) $C_1 = \{110, 11, 10\}$, $C_2 = \{110, 11, 100, 00, 10\}$

b) $C_3 = \{0, 10, 11\}$, $C_4 = \{0, 01, 11\}$

c) $C_5 = \{0, 01, 10\}$, $C_6 = \{0, 01\}$

Problem 2. Consider a set of integers $x_1 < \ldots < x_n$ with associated probabilities $p_1, p_2, \ldots, p_n$. Consider a static (fixed) binary search tree for the set of these items. Show a lower bound in the form of entropy on the expected length of a search in the tree, where item x_i is searched for with probability p_i. How does the answer changes if we allow a dynamic tree that can be reorganized after each search.

Problem 3. Give an example of a random variable X and an arbitrary code $C : X \to \{0,1\}^*$ such that $E[|C(X)|] < H(X)$. For any random variable X and a code C, show a lower bound on $E[|C(X)|]$ in terms of $H(X)$ which is as good as you can come-up with.

Problem 4. Let n be a positive integer. Consider the following code: each message is a matrix M from $GF[2]^{n\times n}$. The codeword of M consists of M together with parities of each row, each column, and the parity of the parities, i.e., a codeword is from $GF[2]^{(n+1)\times(n+1)}$. How many errors can this code correct? How do you correct the errors?

Problem 5. Let G_1 and G_2 be generating matrices of codes with parameters $[n_1, k, d_1]_q$ and $[n_2, k, d_2]_q$. Find the parameters of the codes generated by the following matrices.

a)

$$\begin{pmatrix} G_1 & 0 \\ 0 & G_2 \end{pmatrix}$$

b)

$$\begin{pmatrix} G_1 & G_2 \end{pmatrix}$$

c)

$$G_1 \otimes G_2 = \begin{pmatrix} a_{1,1}G_2 & a_{1,2}G_2 & \cdots & a_{1,n_1}G_2 \\ a_{2,1}G_2 & a_{2,2}G_2 & \cdots & a_{2,n_1}G_2 \\ \cdots & \cdots & \cdots & \cdots \\ a_{k,1}G_2 & a_{k,2}G_2 & \cdots & a_{k,n_1}G_2 \end{pmatrix}.$$

Here

$$G_1 = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n_1} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n_1} \\ \cdots & \cdots & \cdots & \cdots \\ a_{k,1} & a_{k,2} & \cdots & a_{k,n_1} \end{pmatrix}$$

and $a_{i,j}G_2$ is the matrix G_2 with every entry multiplied by $a_{i,j}$.