$\mathcal{E}x$ - 14-4-2020

14 April, 2020 16:03

$p_1, p_2, \dots p_n$ prob. distr.

$H(p_1, p_2, \dots p_n)$

$\varepsilon \in [0,1]$

$p_1', p_2', \dots p_n'$

$p_i' = \varepsilon p_i + (1-\varepsilon) p_j$

$p_j' = \varepsilon p_j + (1-\varepsilon) p_i$

$H(p_1', \dots p_n')$

$$\underline{H(p_1, \dots p_n) \leq H(p_1', \dots p_n')}$$

$$p_i \lg p_i + p_j \lg p_j \underset{\text{Jensen}}{\geq} p_i' \lg p_i' + p_j' \lg p_j'$$

$f(x) = x \lg x$ concave Fn

$\varepsilon = \frac{1}{2}$

KL-Diverge

p_i, p_j, p_k

3b: $I(X:Y:Z) < 0$ X, Y, Z

$$\overset{\displaystyle\parallel}{\underbrace{I(X:Y)}_{0} - \underbrace{I(X:Y|Z)}_{1}}$$

$Z = X \oplus Y$ $X, Y \in \{0,1\}$

" - "

$$I(X:Y) \text{ "}\leq\text{" } \text{ "}\geq\text{" } I(X:Y|Z)$$

$$H(X,Y) \geq H(X,Y|Z)$$

$\frac{1}{2}$	$Z=0$	$X \in_R \{0,1\}^n$	$Y=X$
$\frac{1}{2}$	$Z=1$	$X \in_R \{0,1\}^n$	$Y=0^n$

X
000
010
001
101

$$I(X:Y) > 0$$

$$I(X:Y|Z) = 0$$

$Z=(X,Y)$

(n, k, d)

minimal distance

length lg # codewords

linear

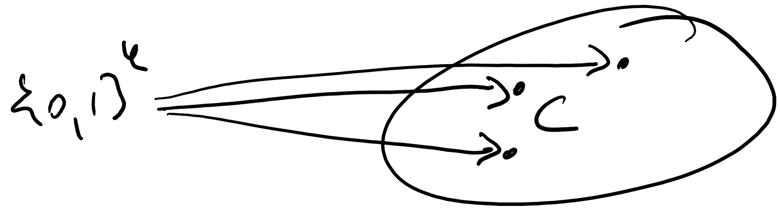
$C_1 = \{00000, 11111\}$ $(5,1,5)$

$C_2 = \{0000, 1111, 0011, 1100\}$ $(4,2,2)$

k bit message $\rightarrow$ n bit ~~message~~ codeword

2^k $\quad$ $2^{\cancel{k}}$ codewords $\subseteq 2^n$

2^k $\longrightarrow$ $\quad$ $|C| \geq 2^k$



$|C| \geq 2^k$

$\boxed{n^2}$ $\boxed{kn}$

Code $(n, \underset{=1}{k}, \frac{3}{4}n)$ $k > 1$

$(n, 1, n)$

allows to correct 0.33%

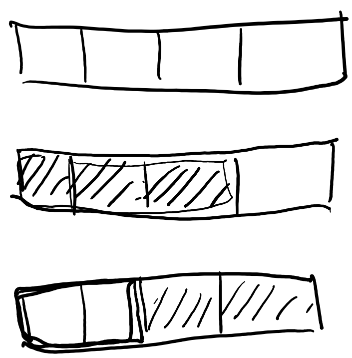
$p = 0.33\%$

$(n, 0.1n, \frac{3}{4}n)$?

$(n, 0.001n, \frac{3}{4}n)$?

$n = 9$ alphabet $= \{0,1\}$

000000000
000111111

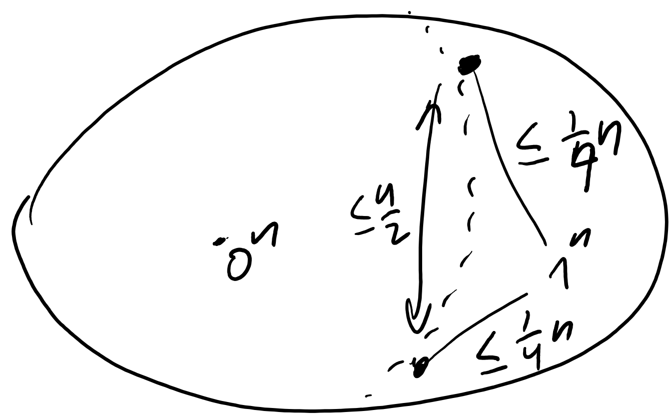


impossible

$$Vol(n, \tfrac{3}{4}n) \geq \tfrac{1}{2} \cdot 2^n$$

$$Vol(n, \tfrac{3}{8}n) \ll 2^n$$

n odd



$$(n, k, \tfrac{2}{3}n)$$

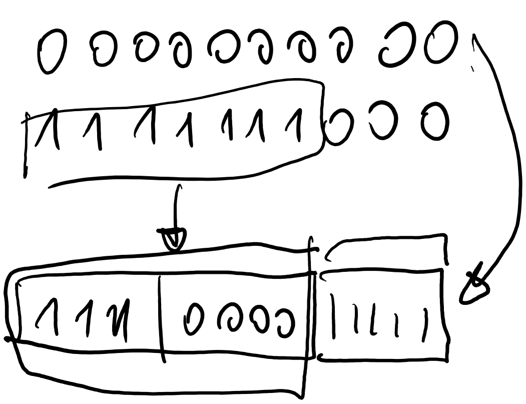
$$|C| \geq 3$$

$$C = \{000,\ 011,\ 110\}$$

$n = 3 \quad d = 2$

$|C| \geq 4$ ~~NO~~

$\frac{2}{2}$



$$C' = \{000,\ 011,\ 110,\ 101\}$$

$(3, 2, 2)$

$|C| \geq 5$ NO

HW - find out $\rightarrow (n, k, (\frac{1}{2}+\varepsilon)n)$ $\varepsilon > 0$

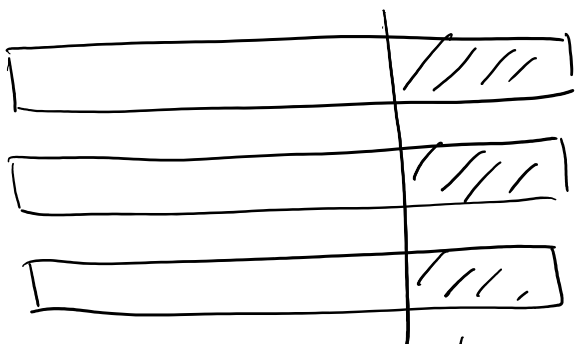
$$2^k \leq 1 + \frac{1}{2\varepsilon}$$

$\varepsilon = \frac{1}{6}$ ≤ 4

$-\ (n, k, \frac{1}{2}n)$ $k = \lg n$

$-\ ((\frac{1}{2}-\varepsilon)n)$ $k = \Theta(n)$

Singleton bound (n, k, d)



$d-1$

$$2^k \leq 2^{n-(d-1)}$$

$$k \leq n-(d-1)$$

$$\cancel{k+d+1}$$

$$k+d \leq n+1$$