NTIN100 Intro to Info Transmission and Processing summer 2020/2021

1st homework assignment - Entropy and mutual information

turn in by April 12th, 2021.

Problem 1.

a) For which probability distributions $p_1, p_2, \ldots, p_n$, the entropy $H(p_1, p_2, \ldots, p_n) = 0$?

b) Let probability distributions $p_1, p_2, \ldots, p_n$ and $p'_1, \ldots, p'_n$ be such that for some i and j, $p'_i = p'_j = \frac{p_i + p_j}{2}$ and for all other k, $p'_k = p_k$. What is the relationship between $H(p_1, p_2, \ldots, p_n)$ and $H(p'_1, p'_2, \ldots, p'_n)$

c) Prove that if $H(Y|X) = 0$ for two random variables X and Y then Y is a function of X.

Problem 2. Let X, Y, Z be random variables, that may or may not be dependent. Prove that

a) $H(X, Y|Z) \geq H(X|Z)$

b) $I(X, Y : Z) \geq I(X : Z)$

c) $I(X : Y|Z) \geq I(X : Z|Y) - I(X : Z) + I(X : Y)$

and decide when equality occurs.

Problem 3. Find (dependent) random variables X, Y, Z such that

a) $I(X : Y|Z) > I(X : Y)$

b) $I(X : Y|Z) < I(X : Y)$

Problem 4. Let X and Y be random variables taking real values $x_1, x_2, \ldots, x_r$ and $y_1, y_2, \ldots, y_s$. Let $Z = X + Y$.

a) Show that $H(Z|X) = H(Y|X)$.

b) Show that if X and Y are independent then $H(X) \leq H(Z)$ and $H(Y) \leq H(Z)$.

c) Find random variables X and Y such that $H(X) > H(Z)$ and $H(Y) > H(Z)$.

d) Under what conditions $H(Z) = H(X) + H(Y)$?