NTIN 100 Intro to Info Transmission and Processing summer 2019/2020

4th homework assignment - Communication complexity

turn in by May 29, 2020.

Problem 1. Let $LESS(x, y) : \{0,1\}^n \times \{0,1\}^n \to \{0,1\}$ be a function that is 1 if and only if $\sum_{i=1}^n x_i \cdot 2^i < \sum_{i=1}^n y_i \cdot 2^i$. Show that its non-deterministic communication complexity satisfies $N^0(LESS) \in \Omega(n)$ and $N^1(LESS) \in \Omega(n)$.

Problem 2. Let $NEQ_m(x_1, \ldots, x_m, y_1, \ldots, y_m) : \{0,1\}^{n\times m} \times \{0,1\}^{n\times m} \to \{0,1\}$ be such that it is one if and only if for all $i \in \{1, \ldots, m\}$, $x_i \neq y_i$. Design a protocol for NEQ_m with error at most 1/4 which communicates at most $O(m + \log n)$ bits. (Alice gets $x_1, \ldots, x_m$, and Bob gets $y_1, \ldots, y_m$.)

Problem 3. Let $MED^*(x, y) : \{0,1\}^n \times \{0,1\}^n \to \{1, \ldots, n\}$ be a function that gives the median of the union of the two sets represented by x and y, that is the median of $\{i \in \{1, \ldots, n\};\ x_i = 1 \text{ or } y_i = 1\}$. Show that the deterministic communication complexity of MED^* is $\Omega(n)$. (Hint: Show a reduction from $DISJ$ to MED^*.)

Problem 4. Consider an undirected graph $G = (V, E)$ with m vertices and n edges. Each subset of the edges of G can be represented by a vector $\{0,1\}^n$, where each coordinate corresponds to an edge of G and indicates whether the edge is present in the subset. Define a code $C_{\text{cut}} \subseteq \{0,1\}^n$ of vectors that represent cuts in G, that is subsets of edges $F \subseteq E$ such that for some subset $S \subseteq V$, $F = \{\{u, v\},\ u \in S\ \&\ v \notin S\}$.

a) Show that C_{cut} is a linear code.

b) Show that if we can efficiently find for each $x \in \{0,1\}^n$ the closest codeword from C_{cut}, then we can efficiently find the largest cut in G. Finding the largest cut in G is so called MAX-CUT problem that is known to be NP-complete.