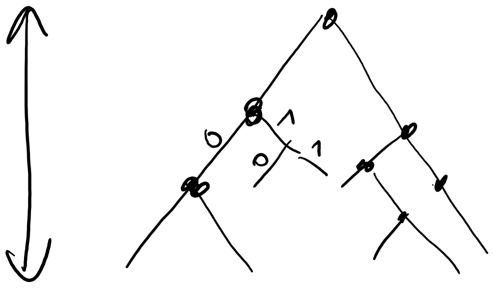
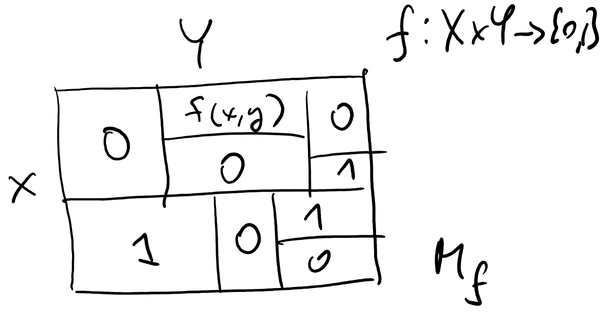
TI-Lecture-19-5-2020

19 May, 2020 13:54



$A \subseteq X \quad B \subseteq Y$

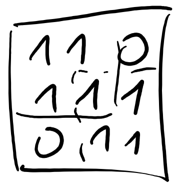
$A \times B$



- monochromatic comb. rectangles

$C(f)$ … the minimum # of $\overset{\text{mono}}{\wedge}$ comb. rectangles that $\underline{\text{cover}}$ M_f

Ex:

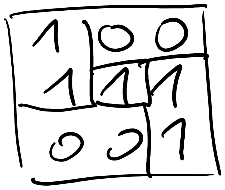


1	1	0
1	1	1
0	1	1

$\wedge|$

$C^D(f)$ … the minimum # of mon. comb. rectangles that $\underline{\text{partition}}$ M_f

Ex:



1	0	0
1	1	1
0	0	1

$\wedge|$

$C^P(f)$ … the minumum # of mono. comb. rectangles given by some protocol for f.

$$C(f) \leq C^D(f) \underset{\neq (\text{Exc})}{\leq} C^P(f) \leq 2^{D(f)}$$

deterministic com. complexity of f

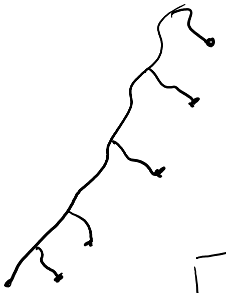
- $\underline{\text{Claim:}}$ $\lg C^P(f) \leq D(f) \leq 2 \cdot \lg_{3/2} C^P(f) \leq 4 \lg C^P(f)$

$O(\lg C^P(f))$

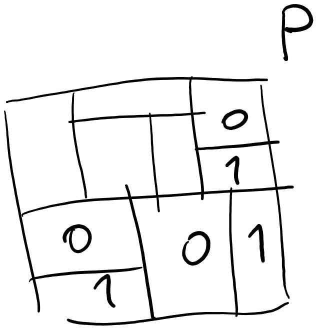
P(.)

$O(\lg C(f))$

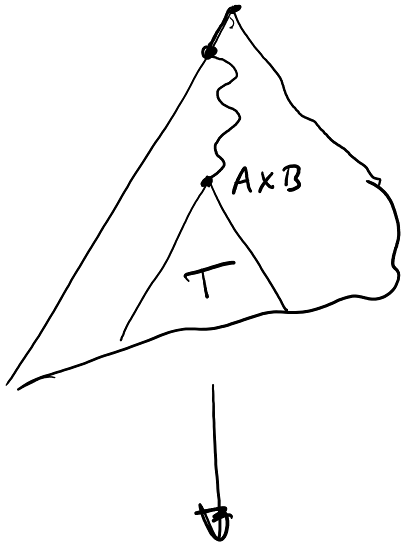
$D(f) \approx \lg C^P(f)$



Pf: $D(f) \leq 2 \cdot \lg_{\frac{3}{2}} C^P(f)$.

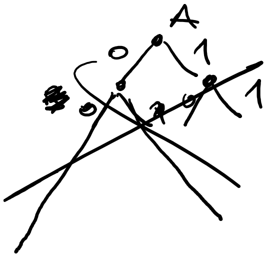


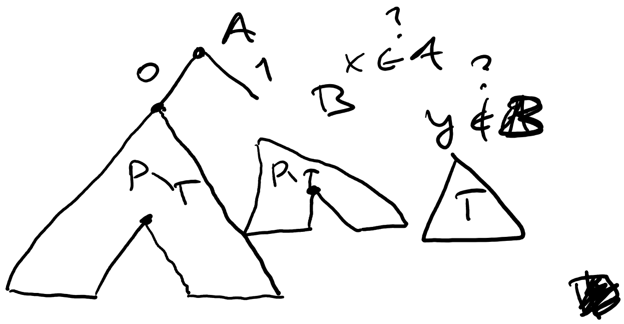
balance the protocol P



$\frac{1}{3}|P| \leq |T| \leq \frac{2}{3}|P|$

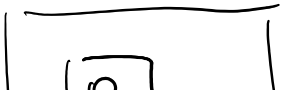
find T by following the larger children from root until you arrive to a vertex defining T.



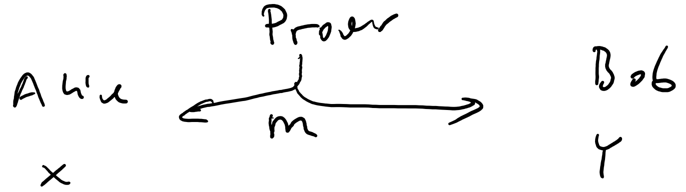


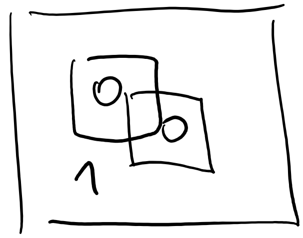
protocols are balanced

C(f) ... monochromatic covs



- nondeterministic communication





Prover wants to convince you that $f(x,y) = 1$

Prover sends a message m to both Alice & Bob. Alice & Bob have to each say whether they got convinced.

Ex: $m = (x,y)$

Ex: $m = y$ Bob confirms $m = y$
Alice confirms $f(x,m) = 1$.

- cost of the protocol $= |m|$.

Ex: P is protocol for f. m = transcript of P on (x,y).

$N(f)$ = max cost of the best nondet. protocol for f.

$N(f) \leq D(f)$

Ex: $NEQ = [x \neq y]$ $D(f) = n+1$
$\lg n \leq N(f) \leq \lg n$

$m = i$ s.t. $x_i \neq y_i$

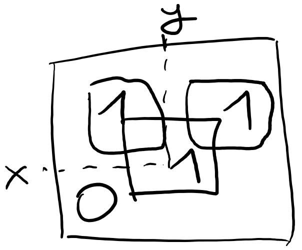
- $D(f) \leq 2^{N(f)} + 1$ $|m| = \ell$
2^{ℓ}

Alice send for each possible message m,

of Prover whether she would accept it.
Bob checks, whether there is a message which both accepts & let Alice know.

- $N(f) = \lg C^1(f)$

  ... min # of 1-monochromatic rectangles to cover all 1's in M_f



$C^0(f)$ ... 0-mono

$C(f) = C^0(f) + C^1(f)$.

"≤"

"≥"

$N(f) \le \lg C(f)$

$D(f) \overset{?}{\approx} N(f)$

$N^{\underline{\underline{1}}}(f)$

Conondeterministic ... Prover is trying to convince you $f(x,y)=0$

$N^{\underline{\underline{0}}}(f)$

- $N^0(f) = \lg C^0(f)$

Ex: $N^0(EQ) = N^1(NEQ) \Big[= N(NEQ)\Big]$

$N(EQ) = n$

$\overbrace{(x,y)} \neq \overbrace{(x',y')}$ m

(x,x), (x',x')
(x,x')

(x,y')
(x',y)

m

Thm: $D(f) \le O(N^0(f) \cdot N^1(f))$.

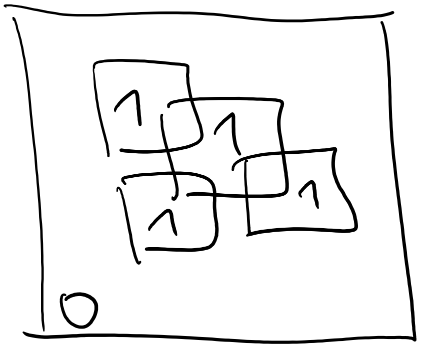
$(D(f) \le O(N^2(f)))$

Pf: protocol with $N^0(f)$ rounds

each round) will require $O(N^1(f))$.

$O(\lg C^1(f))$

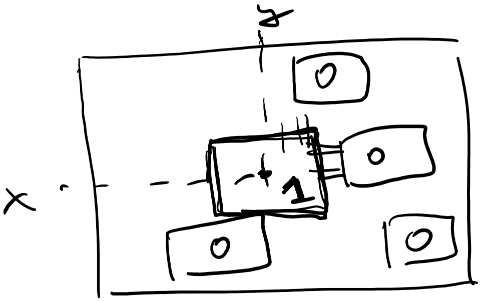
M_f $\lg C^0(f)$



in each round you will find index of 1-mono rectangle

Alice & Bob want to eliminate 0-rectangles that could contain (x,y).

idea: assume $f(x,y) = 1$



each 0-rectangle intersects the 1-rect. either in rows or in columns but not both.

Protocol:

a round: 1) if there are no live 0-rectangles then declare $f(x,y) = 1$.

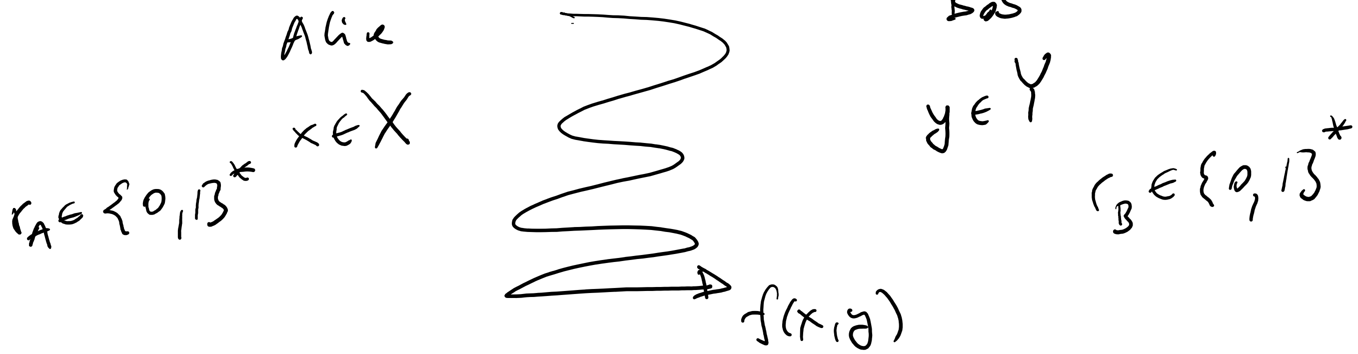
2) If Alice sees a 1-rectangle & its rows which intersects x

intersect at most $\frac{1}{2}$ of live 0-rectangles, she sends ~~the~~ its index to Bob & go-to 1).

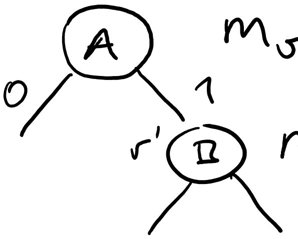
3) If Bob sees a 1-rect. which intersects y & its columns intersect at most $\frac{1}{2}$ of live 0-rectangles, he sends the index of 1-rectangle to Alice. O/w he declares $f(x,y)=0$. ∎

Probabilistic protocols

$F: X \times Y \to \{0,1\}$



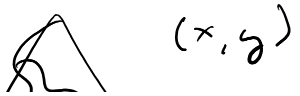
message from A might depend on r_A
messages from B might depend on r_B



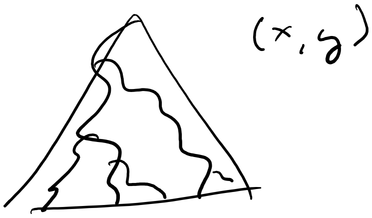
$m_v : \overset{x}{X} \times \overset{r_A}{\{0,1\}^*} \to \{0,1\}$

$m_{v'} : \underset{y}{Y} \times \underset{r_B}{\{0,1\}^*} \to \{0,1\}$

- We say P computes f with error $\le \varepsilon$ if $\forall (x,y) \in X \times Y$ $\Pr_{r_A, r_B}[\text{protocol } P \text{ output } f(x,y) \text{ on } (x,y)] \ge 1-\varepsilon$



$\varepsilon \approx \frac{1}{3}$



$\varepsilon \approx \frac{1}{3}$

- cost of P on (x, y) . . . the average ~~cost~~ cost of P on (x, y)

  → private randomness

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