NTIN 100 Intro to Info Transmission and Processing summer 2018/2019

5th homework assignment - Communication complexity

two days before the exam

Problem 1. Let $GT(x, y) : \{0, 1\}^n \times \{0, 1\}^n \to \{0, 1\}$ be a function that is 1 if and only if $\sum_{i=1}^n x_i \cdot 2^i \geq \sum_{i=1}^n y_i \cdot 2^i$.

a) Show that its deterministic communication complexity satisfies $CC(GT) \leq n + 1$.

b) Show that its deterministic communication complexity satisfies $CC(GT) \in \Omega(n)$.

c) Show that its randomized communication complexity satisfies $R_{1/4}(GT) \in O(\log^2 n)$.

d) Show that its randomized communication complexity satisfies $R_{1/4}(GT) \in O(\log n)$.

Problem 2. Let $MED(x, y) : \{0, 1\}^n \times \{0, 1\}^n \to \{1, \ldots, n\}$ be a function that gives the median of the union of the two sets represented by x and y, that is the median of $\{i \in \{1, \ldots, n\};\ x_i = 1 \text{ or } y_i = 1\}$.

a) Show a deterministic protocol for MED which communicates at most $O(\log^2 n)$ bits and which works whenever x contains only even integers and y contains only odd integers.

a) Show that its deterministic communication complexity satisfies $CC(MED) \in \Omega(n)$. (*Hint:* Use MED to solve $DISJ$.)

Problem 3. *Public vs. private random bits*

a) Let $S_1, S_2, \ldots, S_M \subseteq\in \{0, 1\}^s$ such that each $|S_i| \geq 2^{s-1}$. Show that for all M large enough there is $T \subseteq \{0, 1\}^s$ of size $|T| \leq 1000 \log M$ such that for all $i \in \{1, \ldots, M\}$,

$$\left| \frac{|S_i \cap T|}{|T|} - \frac{|S_i|}{2^s} \right| \leq \frac{1}{12}.$$

(*Hint:* Pick T at random and use Chernoff bound.)

b) Let $f : \{0, 1\}^n \times \{0, 1\}^n \to \{0, 1\}$ be a function with a randomized protocol of cost m with error $< 1/4$ that uses public random bits. Design a randomized protocol for f with error $< 1/3$ that uses private random bits and communicates at most $m + O(\log n)$ bits.

Problem 4. Let $f : X \times Y \to \{0, 1\}$ be a function and $M_f \subseteq \{0, 1\}^{X \times Y}$ be its matrix. Show that if M_f can be partitioned into ℓ monochromatic combinatorial rectangles then $\text{rank}(M_f) \leq \ell$. Show that $CC(f) \geq \log \text{rank}(M_f)$.