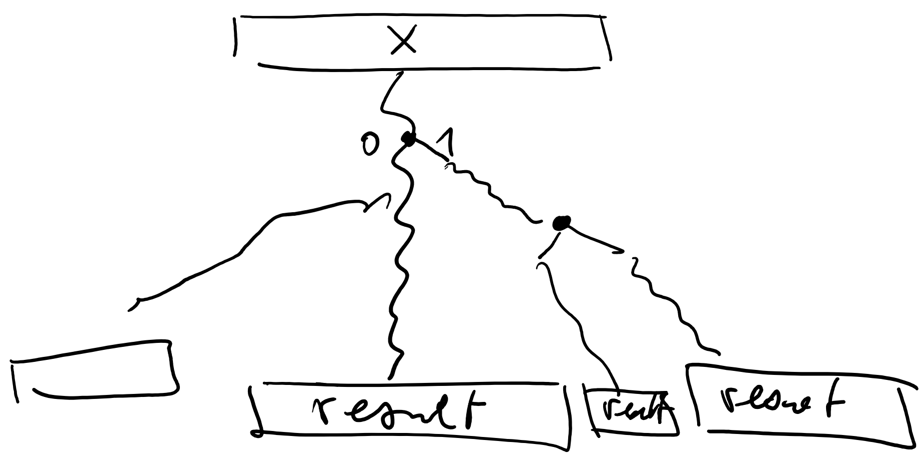
CC - 18.3.2020

18 March, 2020 10:30

This is the first lecture

Alternating computation

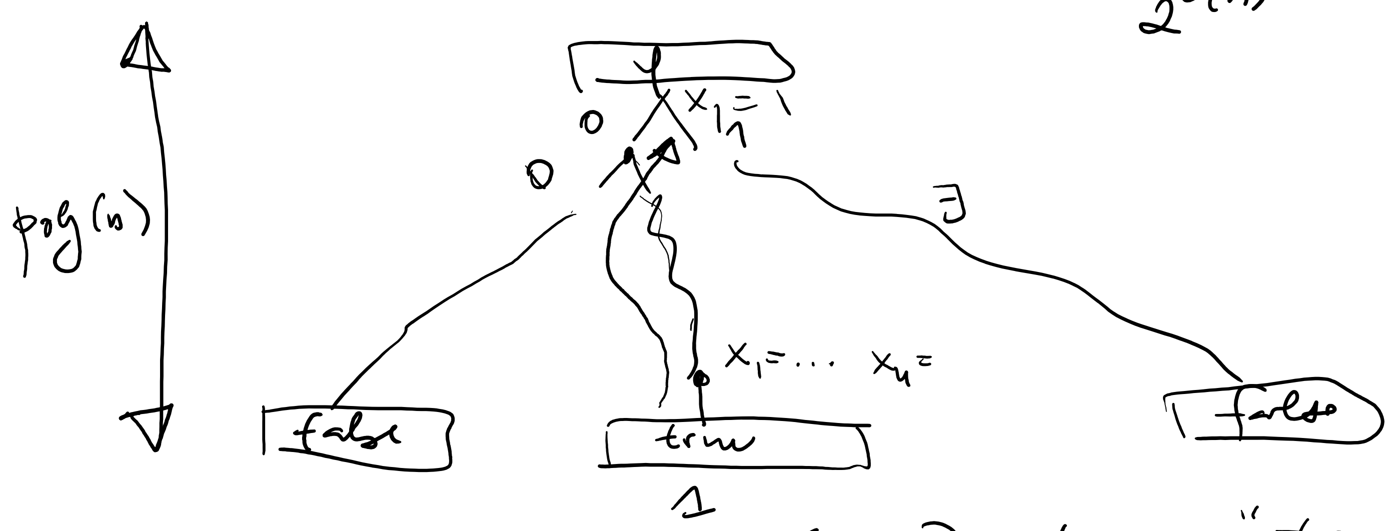
nondeterministic computation



x := random bit
nondeterministic

φ ... boolean formula $(x_1 \vee x_2) \& (x_7 \vee x_5) \ldots$

is it satisfiable? ... SAT $2^{O(n)}$



φ is satisfiable $\Leftrightarrow$ $\exists$ a branch "true"

Def: Algorithm A accepts its input x iff ∃ a branch which accepts.

NP ... problems (L) for which there is a nondeterministic algorithm running in poly time which accepts exactly strings from L.

Ex: • SAT is NP, Hamiltonian cycle

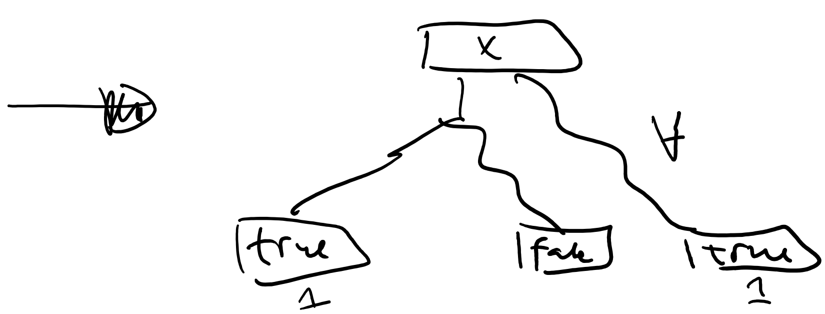
• TAUT = { φ : φ is a Boolean formula which is true for all assignments to its variables }

$$\varphi = x_1 \vee \neg x_1 \qquad \varphi \in TAUT$$

TAUT $\in$ NP?

conondeterministic computation

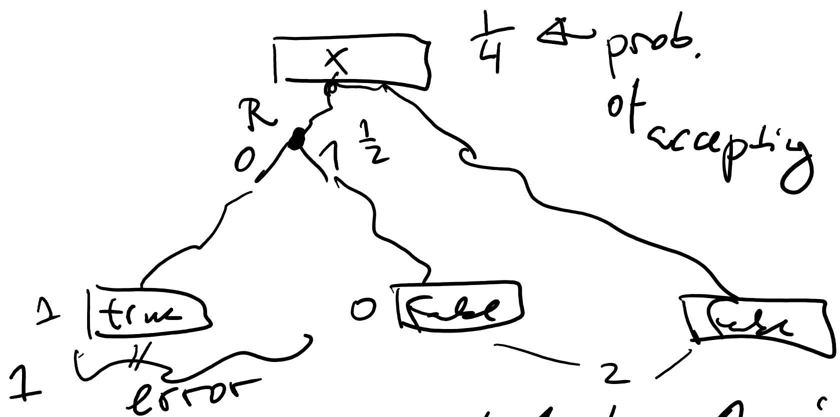
– nondeterministic algorithm A



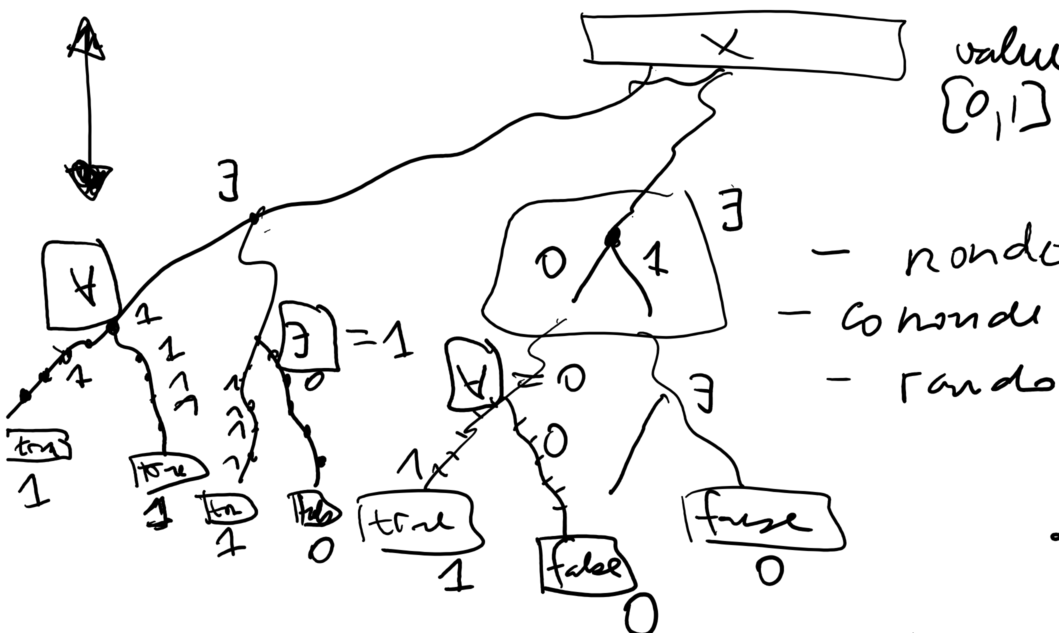
Def: conondeterministic machine A accepts input x iff all computation paths accept x

TAUT ... ∃ conondet. machine runs in p(n) time and accepts x iff $x \in TAUT$

randomized computation - $\underline{\underline{A}}$



x is accepted by A iff majority of computations accepts x.



- nondeterministic ∃
- conondeterministic ∀
- randomized R

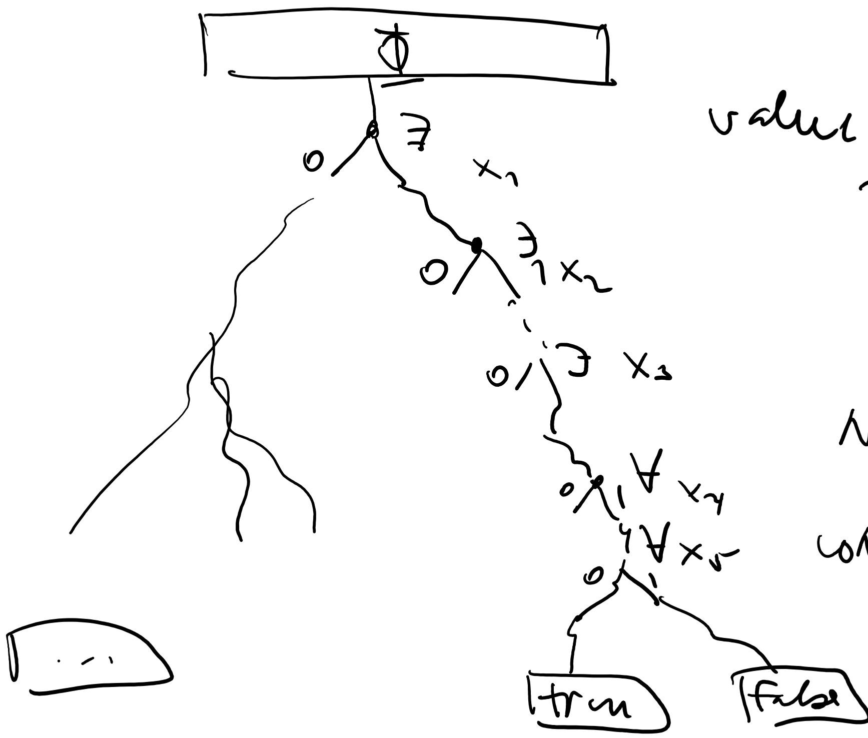
{0,1} [0,1]

generalized alternating machine A accepts x iff the tree is accepting iff the tree is accepting

		00	01	10	11
$\forall$	min value	0	0	0	1
$\exists$	max value	0	1	1	1
(R	average value	0	$\frac{1}{2}$	$\frac{1}{2}$	1)

$$\overline{\underline{\Phi}} = \underline{\exists} \overbrace{x_1 \exists x_2 \exists x_3} \overbrace{\forall x_4 \forall x_5} \varphi(x_1, x_2, x_3, x_4, x_5)$$

is this formula true?



value of the tree $\equiv$ $\underline{\Phi}$ true

SAT

NP $\exists x_1 \exists x_2 \cdots \varphi()$

TAUT

coNP $\forall x_1 \forall x_2 \cdots \varphi(x_1 \ldots)$

QBF

$\exists x_1 \forall x_2 \exists x_3 \cdots$

this can be decided in alternating poly-time

Thm: In PSPACE we can decide whether an alternating machine running in poly-time accepts/rejects.

alt. TM A input x

A accepts x or not

- depth first search of the computation tree.

- true even for generalized TM's.

$GATIME(t(n)) = \{L;\ L$ is a language for which there is a generalized alternating TM running in time $t(n)\}$

$SAT, TAUT \in GATIME(n^2)$

$GAP = \bigcup_k GATIME(n^k + k)$

- $GAP \subseteq PSPACE$

  " $\supseteq$ "

<u>Claim</u>: $PSPACE \subseteq GAP$

QBF is complete for PSPACE

$L \in PSPACE$ ... $\exists$ algorithm running in <u>space</u> n^{100}

$\exists$ polytime procedure $\underline{f}$

$\underline{\underline{x}} \longrightarrow \underline{\underline{f(x)}}$ .... quant. Bool Fce.

$x \in L \iff f(x)$ is true

QBF $\in$ GAP

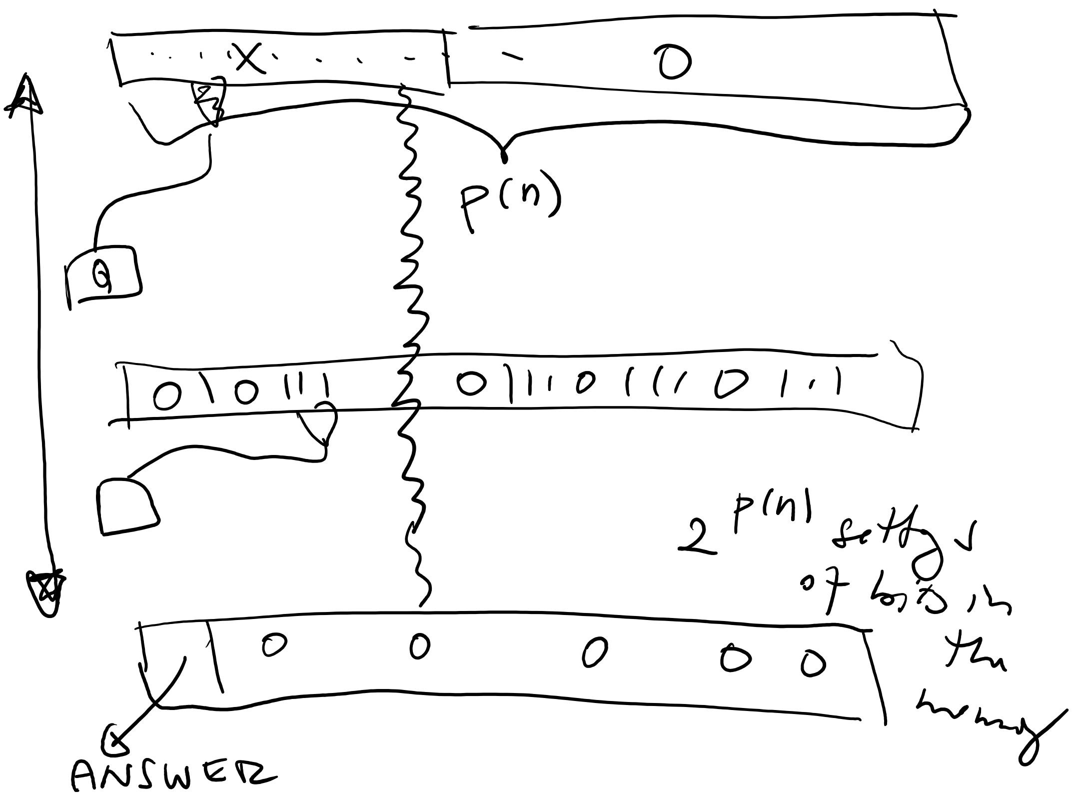
- QBF is complete for PSPACE

$L \in$ PSPACE

L ... (deterministic) algorithm A which decides A & runs in polynomial space

Ex: $p(n) = n^{100}$

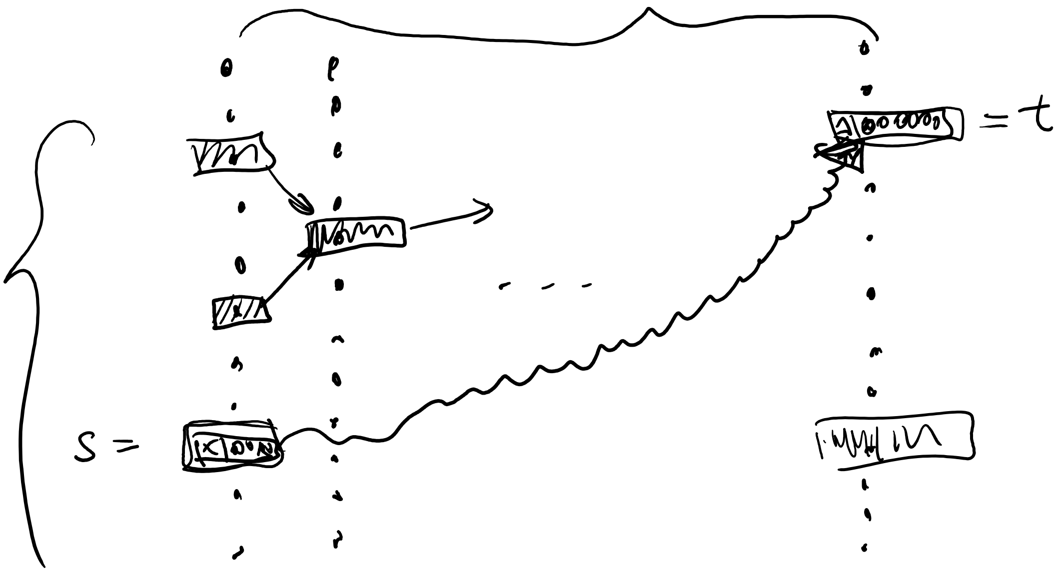
$|Q| \cdot p(n) \cdot O(2^{p(n)})$



PSPACE $\subseteq$ EXP ... exponential time

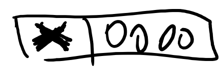
W.L.O.G. (without loss of generality) calculation takes exactly $|Q| \cdot p(n) \cdot 2^{p(n)}$ steps

Graph



$|Q| \cdot p(n) \cdot 2^{p(n)}$ vertices

$|Q| \cdot p(n) \cdot 2^{p(n)}$



Q: Is there a path from s to t?

$\boxed{x} \longrightarrow G, s, t \xrightarrow{QBF} \Phi$

$x \in L$ iff $s \rightsquigarrow t$ iff Φ true

size $\approx p(n)^2$