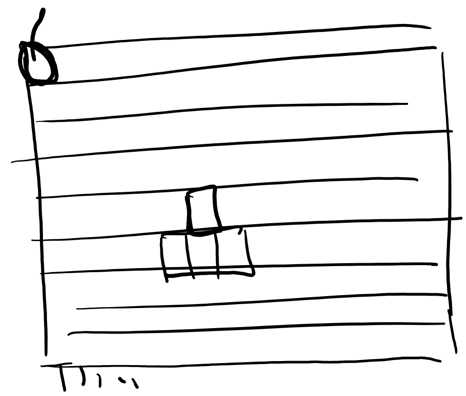
CC-Lecture-15-4-2020

15 April, 2020 10:35

Boolean Circuits

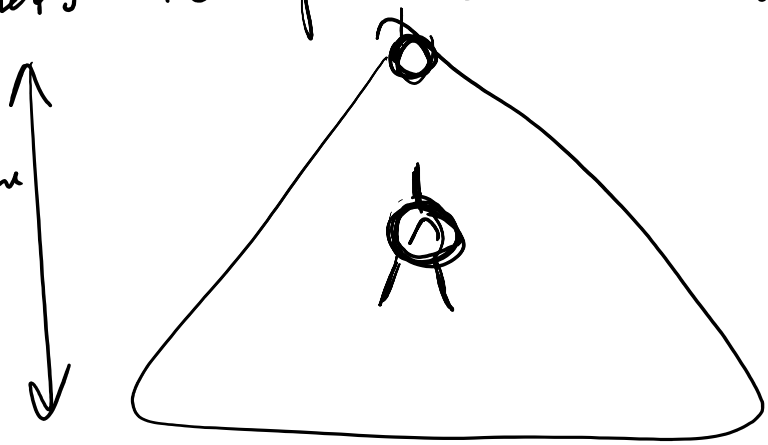
- poly-size ckt's can compute $\forall$ in P.



$t(n)$

depth $\simeq$ time

ckt's $\approx$ parallel model of computation



depth $\simeq$ time

processors $\approx$ gates

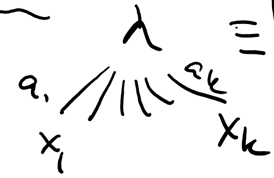
CPU
chip

size, depth

gate - AND, OR, $\neg$

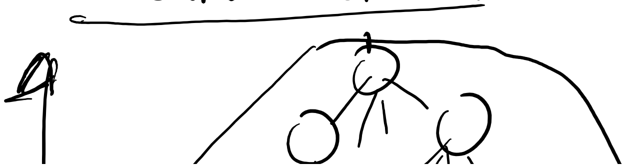
THR, MAJ

MAJ

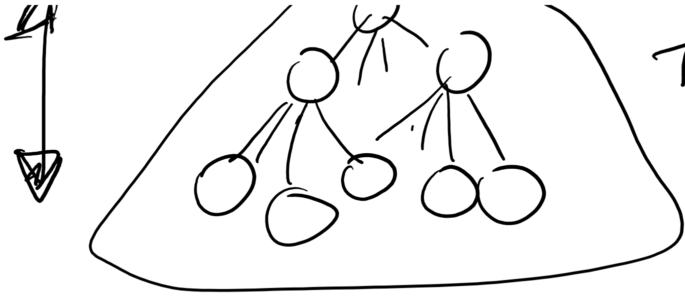


$= \left[\sum_i x_i \geq \frac{k}{2} \right]$

Neural networks

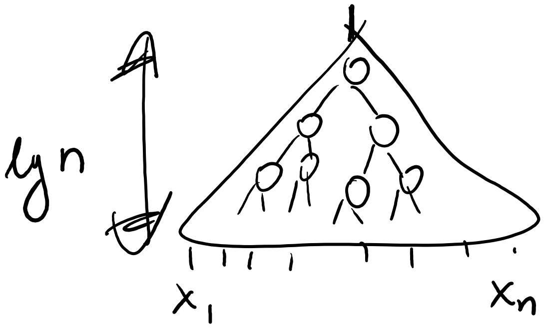


Thr $\left[\sum_i^k a_i x_i \geq \theta \right]$



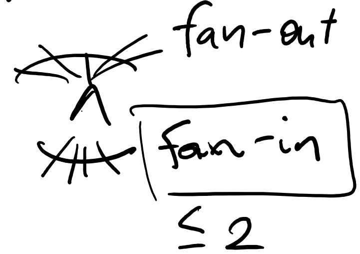
Thr $\left[\sum_{i=1}^{n} a_i x_i \geq \theta\right]$

a_i



$\wedge$ - binary AND, OR

$\wedge$ unary NOT



- depth $O(\lg n)$, poly-size, binary AND, OR, NOT

NC^1 ... class of fcn's computed by families of ckt's

NC^2 ... $O(\lg^2 n)$ - depth —//—

NC^k ... $O(\lg^k n)$ - depth —//—

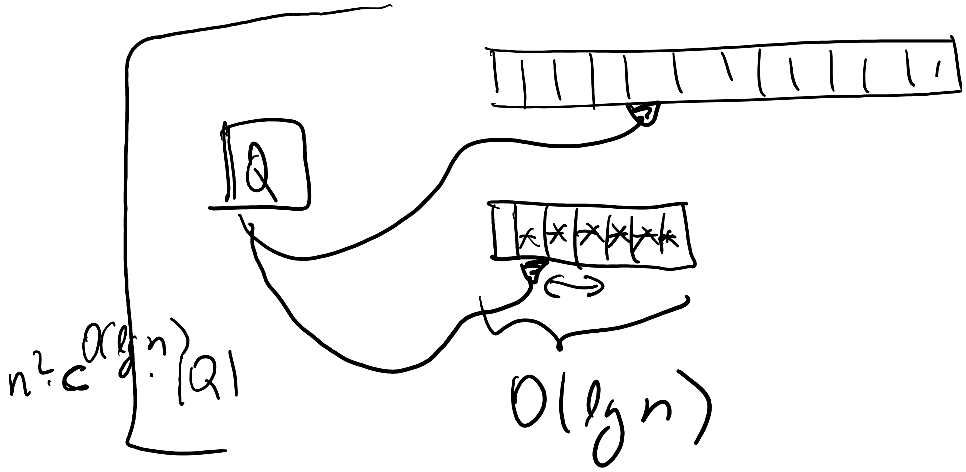
- $P \subseteq NC^1$???
- $NP \subseteq NC^1$??? "can we parallelize sequential computation?"
- $EXP \subseteq NC^1$???

- $NP \not\subseteq PSIZE \Rightarrow NP \not\subseteq NC^1$

$NC^1 \subseteq PSIZE$

- $LOG = DSPACE(\lg n)$, $NL = NSPACE(\lg n)$

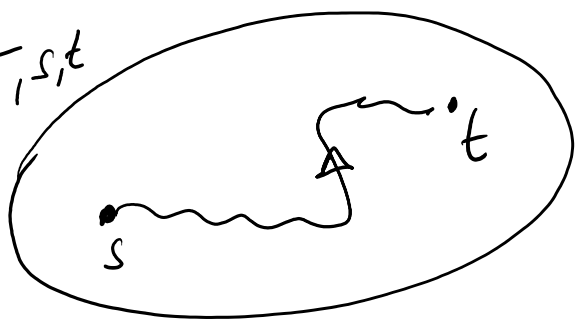
$LOG \subseteq NC^2$ $\qquad$ $NL \subseteq NC^2$



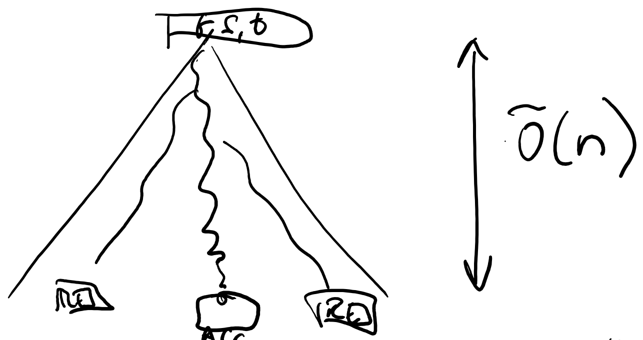
input R/O

work-space $O(\lg n)$ R/W

Input: G, s, t



Q: Is there a path from $s \to t$?



$O(n)$ ... space alg.

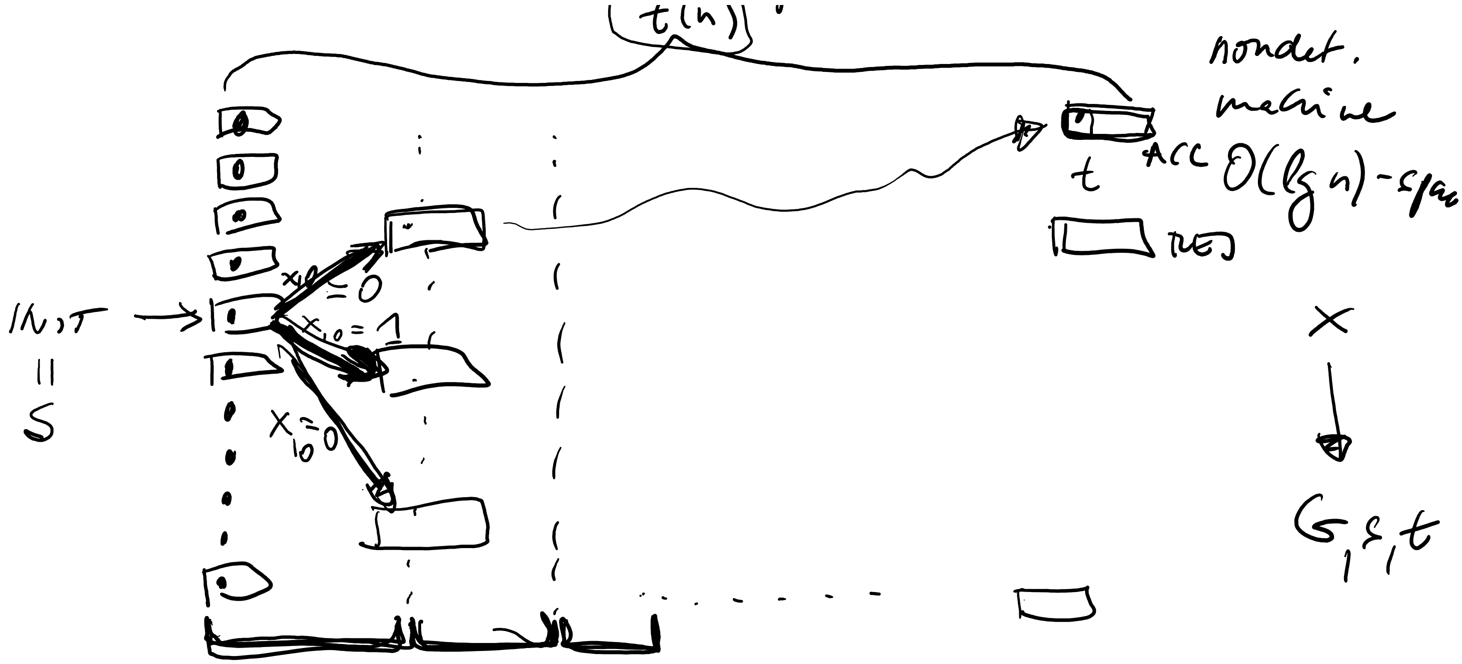
$O(\sqrt{n})$ ... space alg? (and poly-time) deterministic

nondeterministic: $O(\lg n)$-space poly-time

= STCONN ... complete problem for NL

$t(n)$

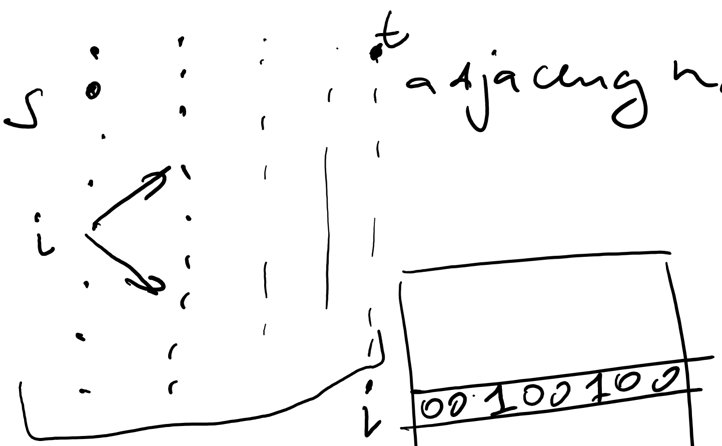
nondet.



$c^{c' \lg n} = n^{c' \cdot \lg c} = n^{O(1)} \cdot n^2 \cdot |Q| = \mathrm{poly}(n) = t(n)$

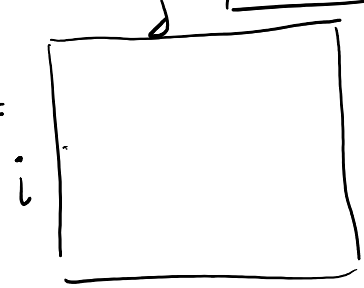
- layered graph, $t(n)$ repeated first layer

adjacency matrix of the first layer A_G



$= A_G$

$A_G^2 =$



$(A_G^2)_{i,j}$ = # ~~the~~ walks of length 2 between i & j

$(A_G^{t(n)})_{s,t} = 0$?

$A_G \times A_G$ (over Boolean-~~fields~~)

$A_G \times A_G$ (over Boolean-~~semiring~~ semiring)

$(A_G \times A_G)_{ij} = 0/1$ $\quad i \to j$ of lgt 2

$+ \ldots$ OR

$\cdot \ldots$ AND

reo

$\left(A_G^{t(n)}\right)_{s,t}$ $\qquad t(n)$ - times

$((A_G \times A_G) \times (A_G \times A_G)) \cdot (\ldots \times A_G$

$A_G^2 \times A_G^2 \times \ldots \quad A_G^2$

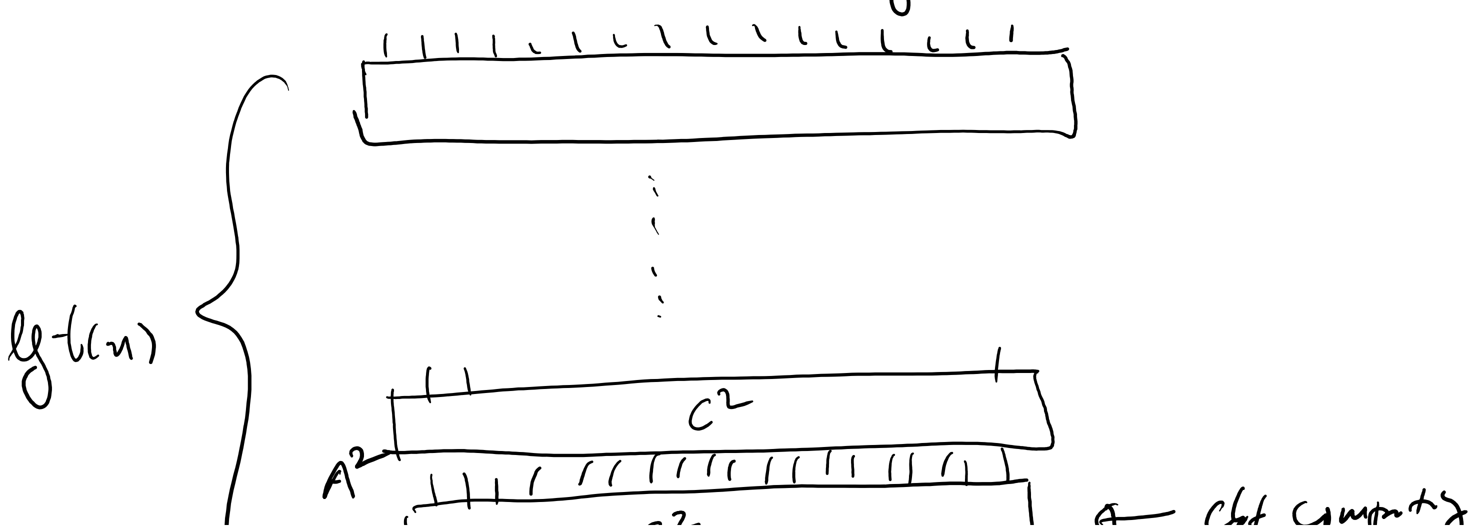
$\underbrace{\left(\left(\left(A_G\right)^2\right)^2\right)^{\cdots}}_{\lg t(n)\ \text{-times}}$

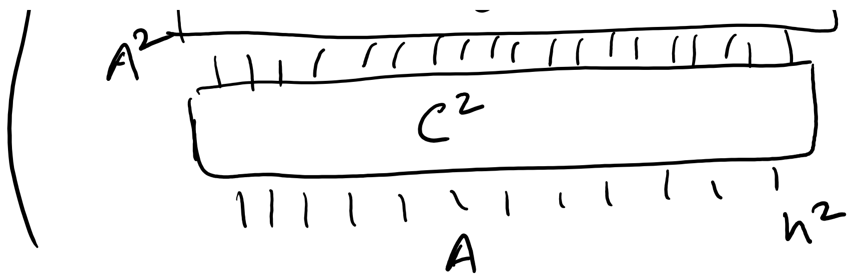
$A^{\overbrace{2\cdot 2\cdot 2\cdot 2\cdot}^{\lg t(n)}} = t(n)$

$\mathrm{pol}(n) = t(n)$

$\lg t(n) \approx \underline{O}(\lg n)$

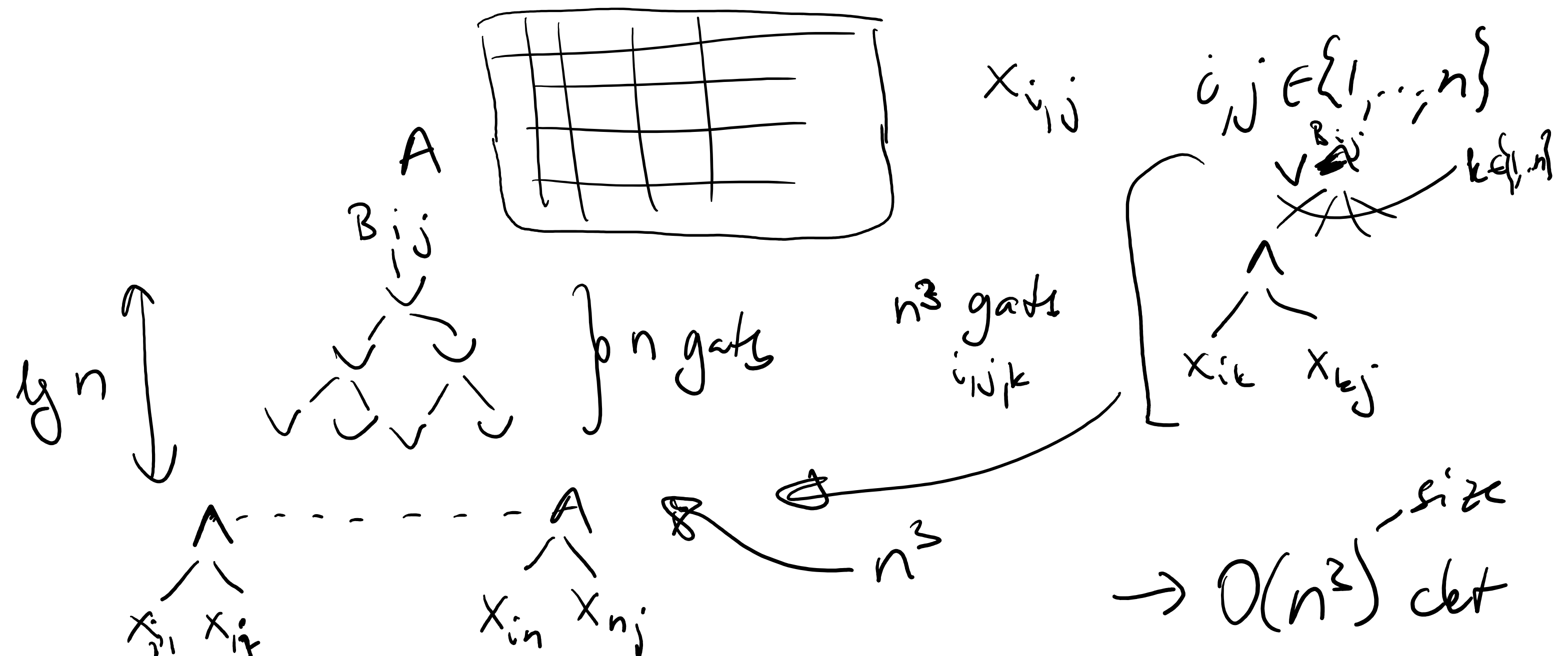
ckt : A_G $\quad \{0,1\}^{n \times n} \to \lg(t(n))$ times





ckt computing a square of the matrix

C^2 $\quad B = A^2 \quad B_{ij} = \underset{k\in\{1,..,n\}}{OR} (AND\ x_{ik}\ x_{kj})$



$\rightarrow O(n^3)$ ckt

depth $1 + \lg n$

C^2 ckt

ckt for $A_G^{t(n)}$ ... depth $O(\lg^2 n)$

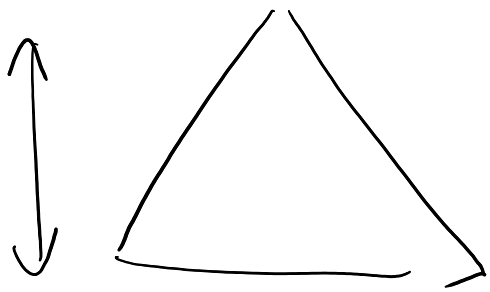
size $O(n^3 \cdot \lg n)$

→ designed a NC^2 for matrix powering over Boolean semiring

$\Rightarrow$ • $NL \subseteq NC^2$ ... parallelize any problem solvable in NL

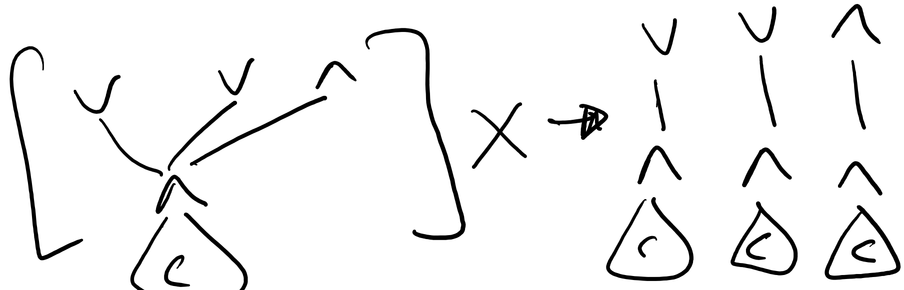
• $NL \subseteq NC^1$?

$d = O(\lg n)$



binary AND, OR, NOT

Boolean formulas

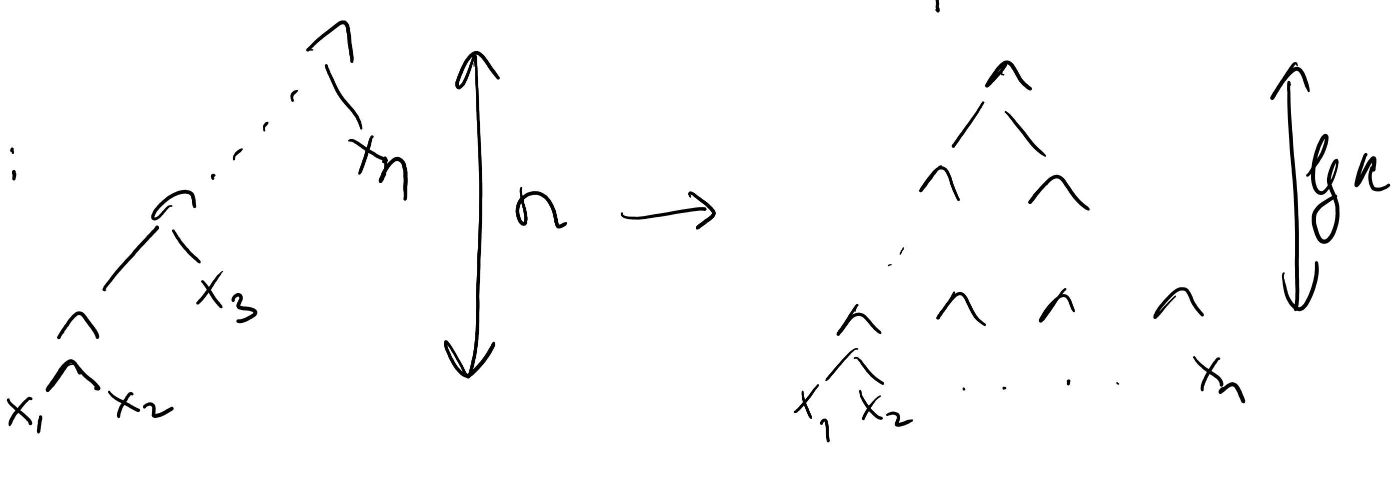


duplicate sub-circuits

$\Rightarrow$ formula → depth $d = O(\lg n)$

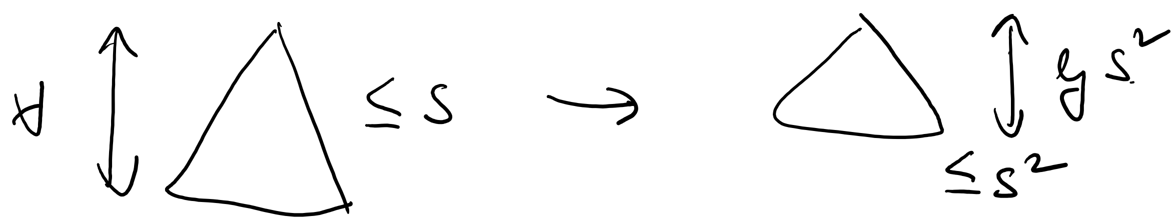
size $\leq 2^d = poly(n)$

Ex:

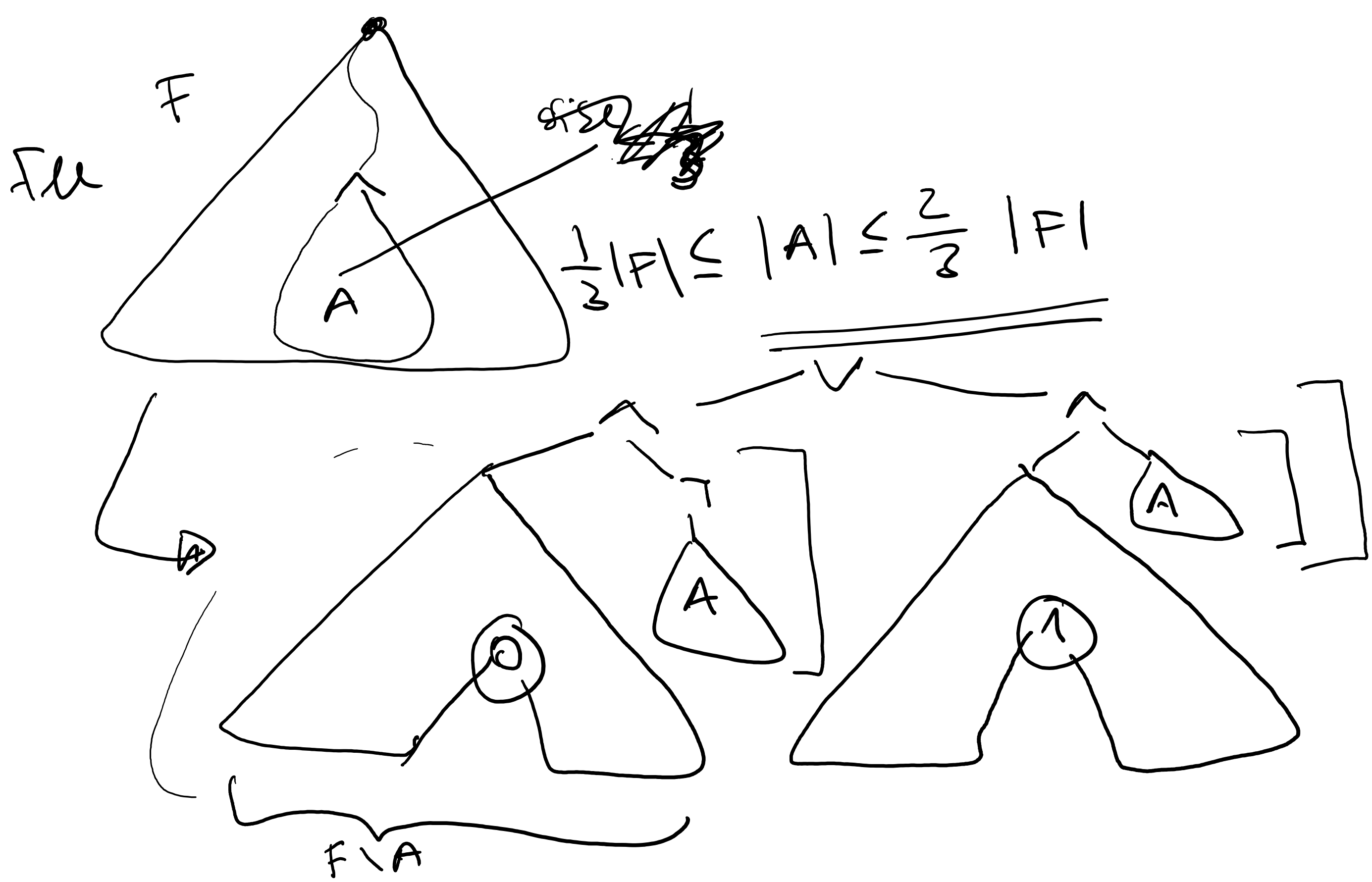


Formula of size S "poly(n)" $\longrightarrow$ $\updownarrow \lg S^2$

arbitrary depth



Balancing formulas



$\Rightarrow$ depth of the formula $\le \frac{2}{3}$ $\quad |F| \to 4$

depth $2 \lg |F|$ $\qquad \lg_{\frac{3}{2}} |F| \le 2 \lg_2 |F|$ (Exe) dpth $2 \lg |F|$

size $\leq 2^{2 \lg |F|} \leq |F|^2$

(Exe)

$\lg_{\frac{3}{2}} n = \frac{\lg n}{\lg \frac{3}{2}}$

- poly-size formula $\rightarrow$ poly-size, $O(\lg n)$-depth formula

$\Rightarrow$ $NC^1 \equiv$ poly-size formulas