CC-Lecture-13-5-2020

13 May, 2020 10:36

- ckt's
  - depth
- fle's - size, depth
- polynomial
  - degree
  - size of coef's

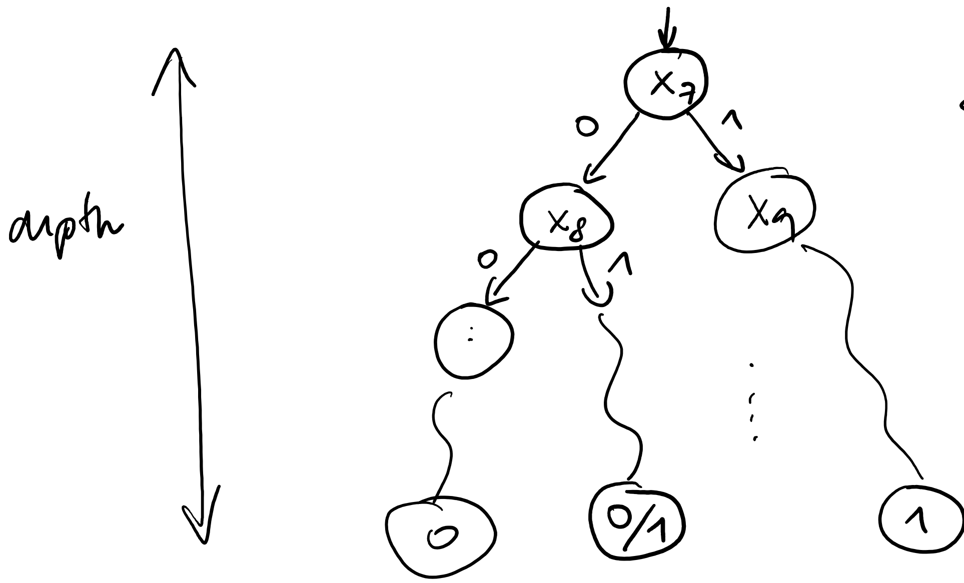
$f : \{-1,1\}^n \to \{0,1\}$

$\{0,1\}^n \to \{0,1\}$

GF[

- decision trees, branching pgm's

$f : \{0,1\}^n \to \{0,1\}$ $\quad x_1, x_2, \ldots x_n$ input variables

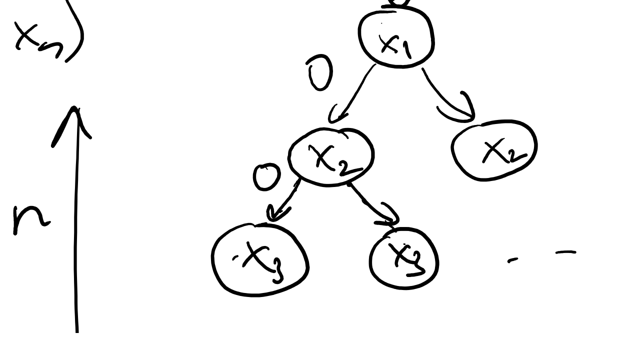


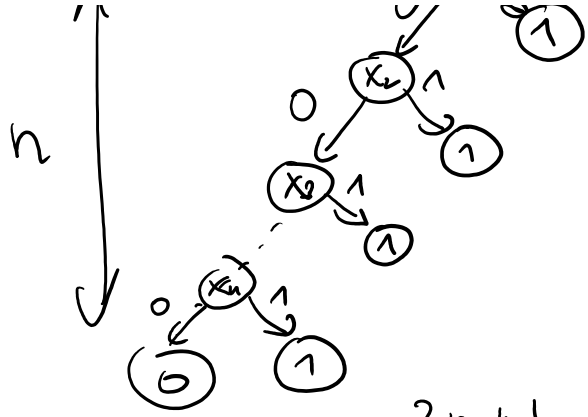
decision tree
- each node is labelled by a variable
- follow the path from root consistent with the variable assignment
- leaves give output value.

size

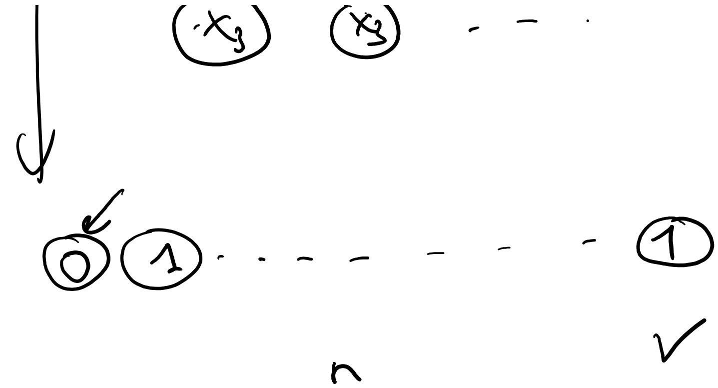
Ex: 1) $f(x) = 1 \quad \forall x \in \{0,1\}^n$

2) $f(x) = OR(x_1, \ldots x_n)$



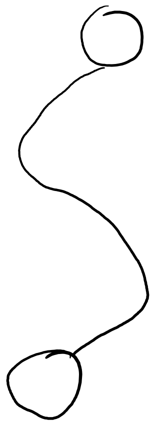


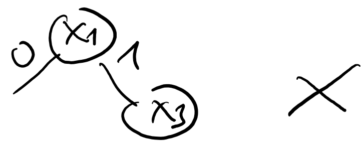
$2n+1$



2^n

$\geq n$



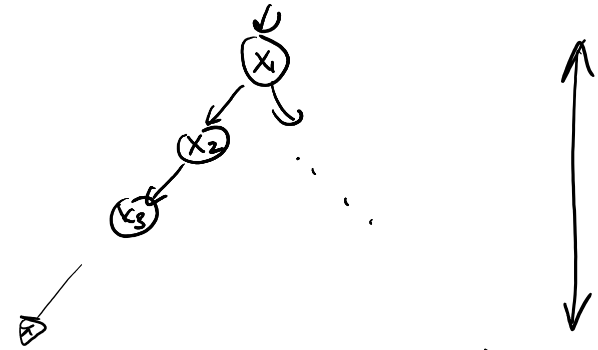


on $x_1 = x_2 = \quad = 0$ one must read all variables

· degree polyn $\approx$ depth of the decision tree

<u>Ex</u>: $XOR(x_1 \dots x_n) = \sum x_i \bmod 2$
PARITY

$x \in \{0,1\}^n$



depth n

$\geq n$

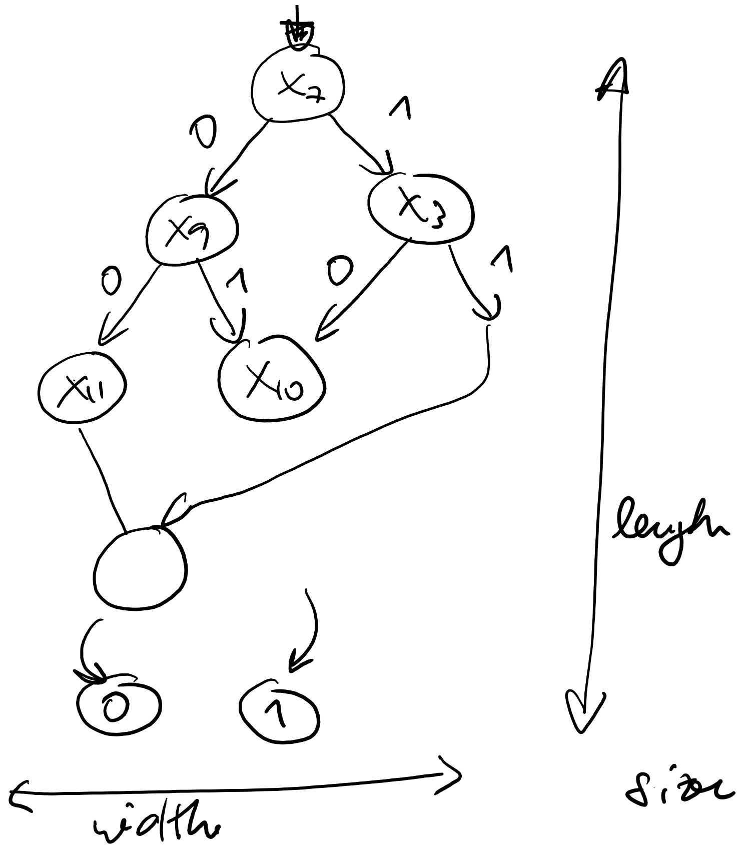
2^n

A

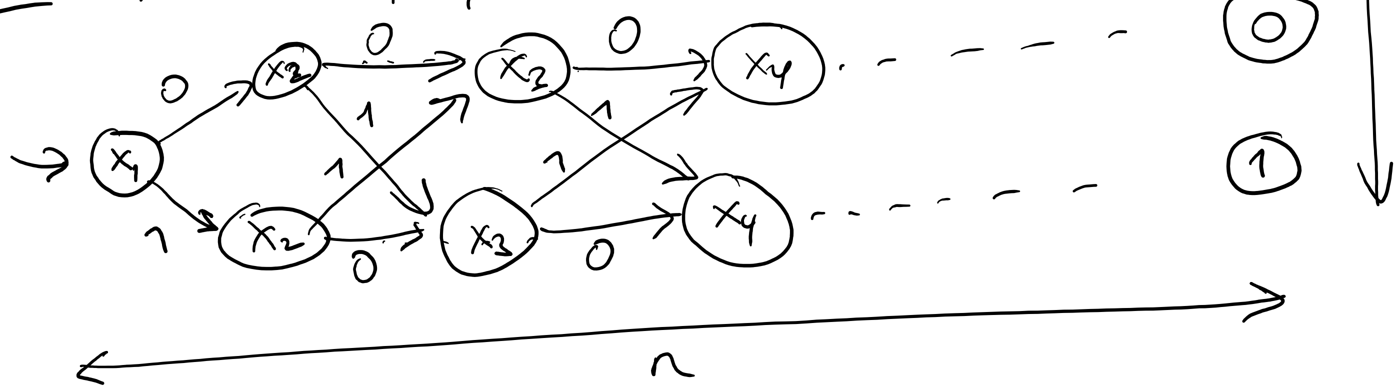
<u>branching programs</u>

branching programs

- directed acyclic graph
- each node is labelled by a variable
- follow a path from root until a sink is reached labelled by the output value



EX: PARITY$(x_1, x_2, \dots x_n)$

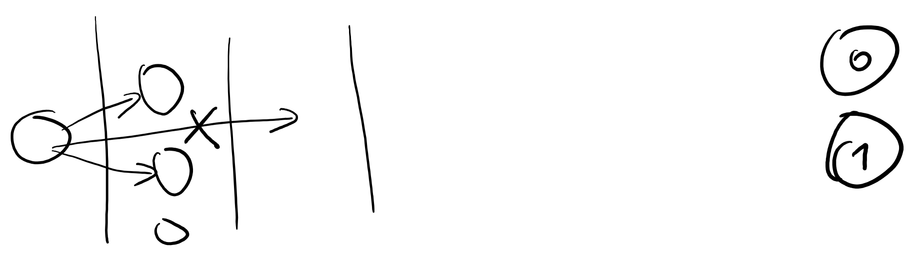


$2n+1$ … size width … 2

width

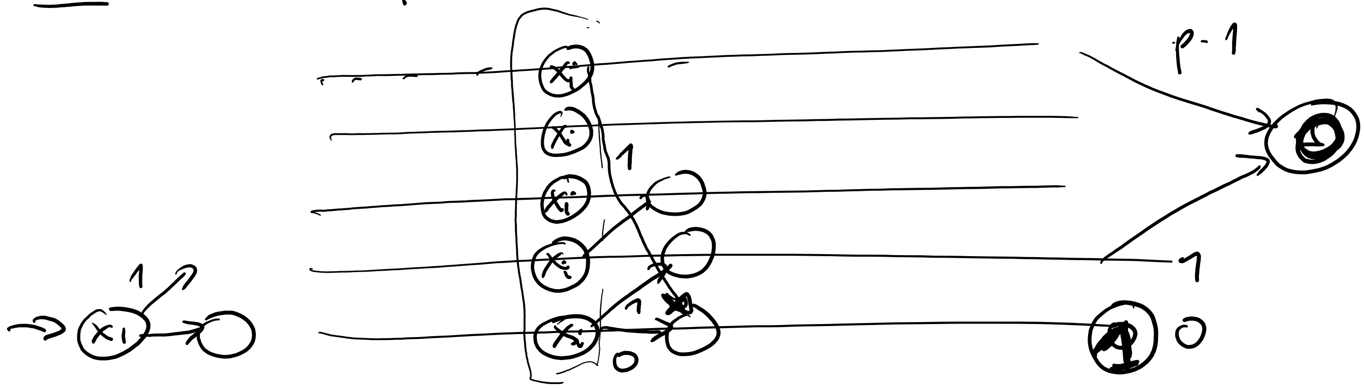
layer the b.p.

width = size of the largest layer



largest layer

Ex: $MOD\text{-}p(x_1 \dots x_n) = [\,p \mid \sum x_i\,]$



width p   size $\simeq pn$ length n

smaller width?

- width $\simeq$ memory size   length $\simeq$ time

width 2^s $\simeq$ space s

- bp of size S you can evaluate it in space $O(\lg S)$

$S = n^{O(1)}$ $O(\lg n)$

- f is computable by a polynomial size branching pgm's then it is computable in L/poly.
  - L → log-space
  - /poly → poly-size advice

- poly-size ckt's = P/poly
- poly-size b.p's = L/poly ("non-uniform log-space")

- $f \in L/poly \Rightarrow f$ poly-size b.p.'s ?

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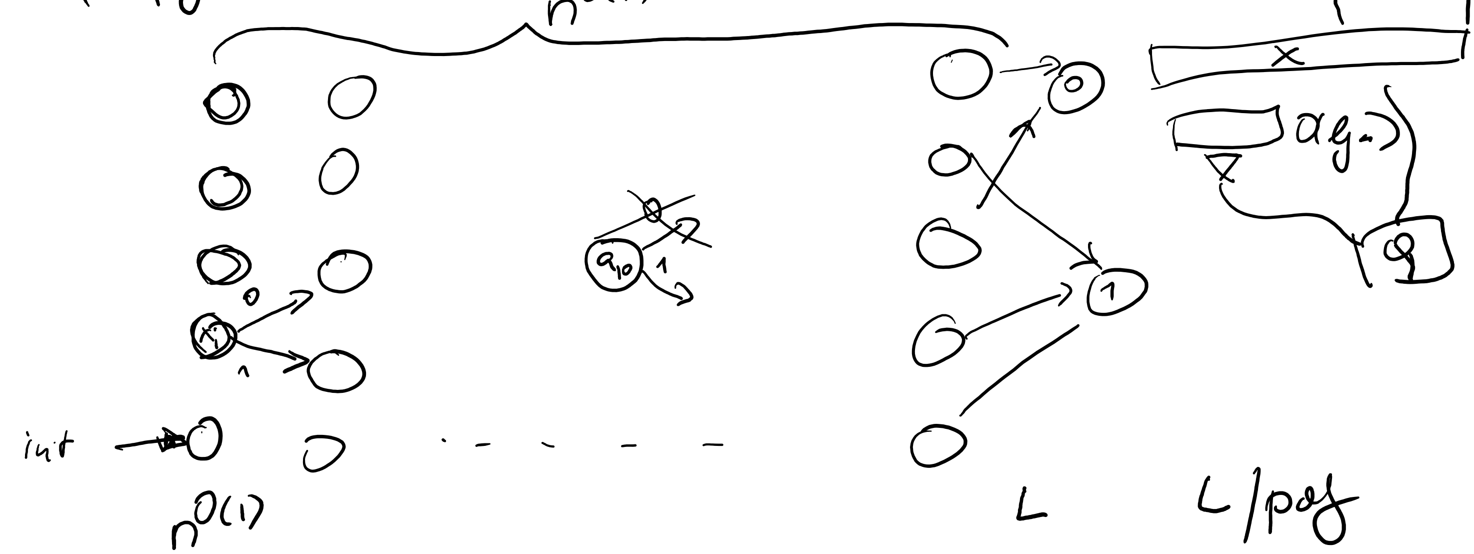
- f poly-size b.p.'s $\Rightarrow f \in L/poly$.



- NEXP $\subseteq$ poly-size b.p.'s ?
- NP $\subseteq$ poly-size b.p.'s ?

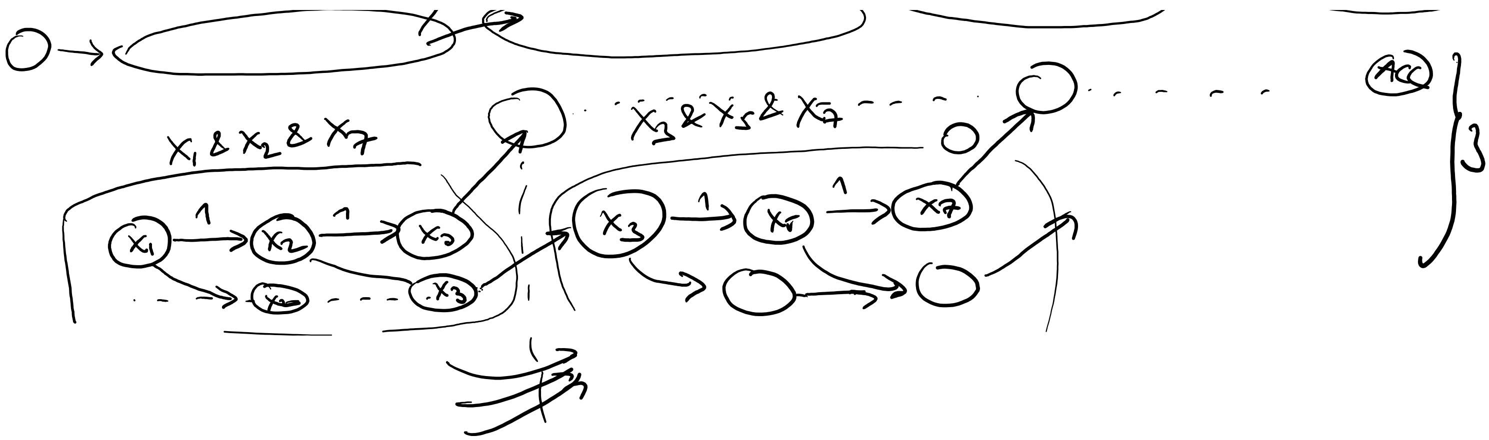
conjecture: No
$\Downarrow$
$NP \neq L$

- $NP = L$?
- $P = L$?
- $P \subseteq$ poly-size b.p.'s ?

- PARITY$(x_1, \dots x_n)$ ... width of the of b.p. ?

DNF $\longrightarrow$ branching pgm of width 3.

$(x_1 \& x_2 \& x_7) \vee (x_3 \& x_5 \& \neg x_6) \vee \dots$



⇝ CHEAT ... exponentially long b.p.

Q: B.p. for MOD-p of width $< p$ & polynomial size?

Barrington '87: MOD-p can be computed by width 5 b.p. of poly-size.

NC^1 by width 5 b.p. of poly-size.

• Ben-Or & Cleve '88 generalization of ↑.

permutation b.p. of width 5