CC-Lecture-25-3-2020

25 March, 2020 10:23

- QBF is PSPACE complete

$$\exists x_1 \forall x_2 \exists x_3 \exists x_4 \dots \varphi(x_1, x_2, \dots x_n)$$

is it true. $\in$ PSPACE

Any $L \in$ PSPACE

$x \in \{0,1\}^n \rightsquigarrow \Phi$

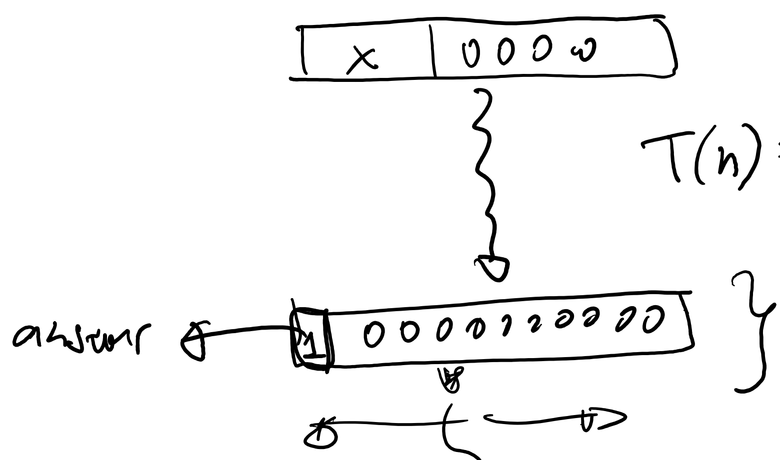
$x \in L$ Φ true

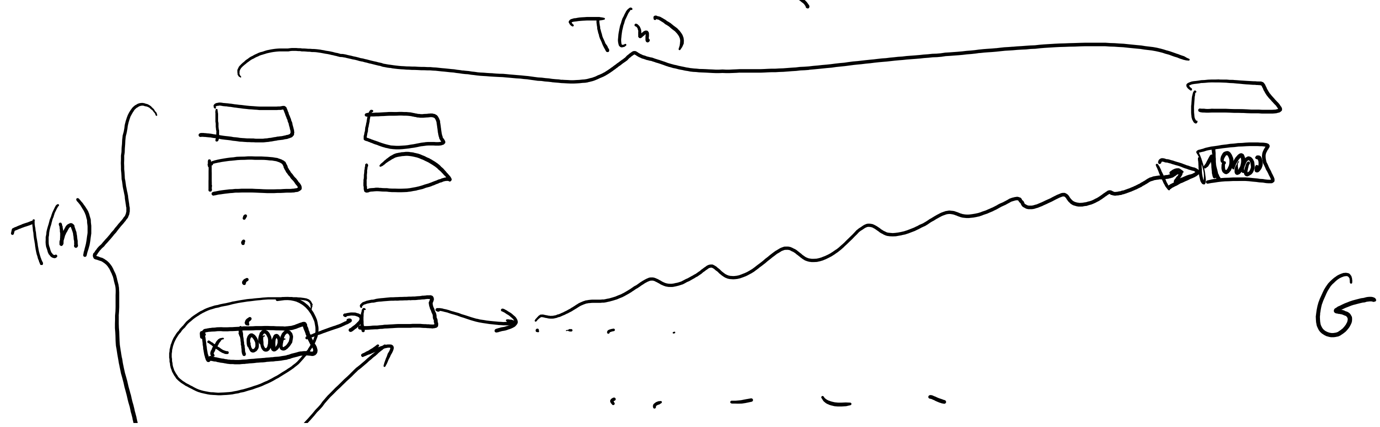
$x \notin L$ Φ is false

$\exists$ TM which runs in space $p(n)$ and decides L.

$$T(n) = O(|Q| \cdot p(n) \cdot 2^{p(n)})$$

steps



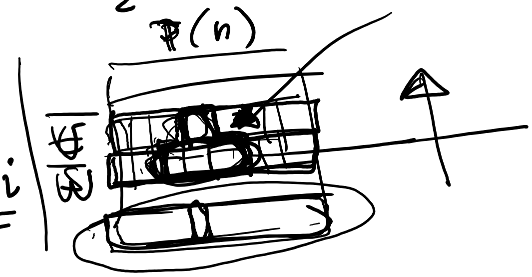


all possible configuration of the machine

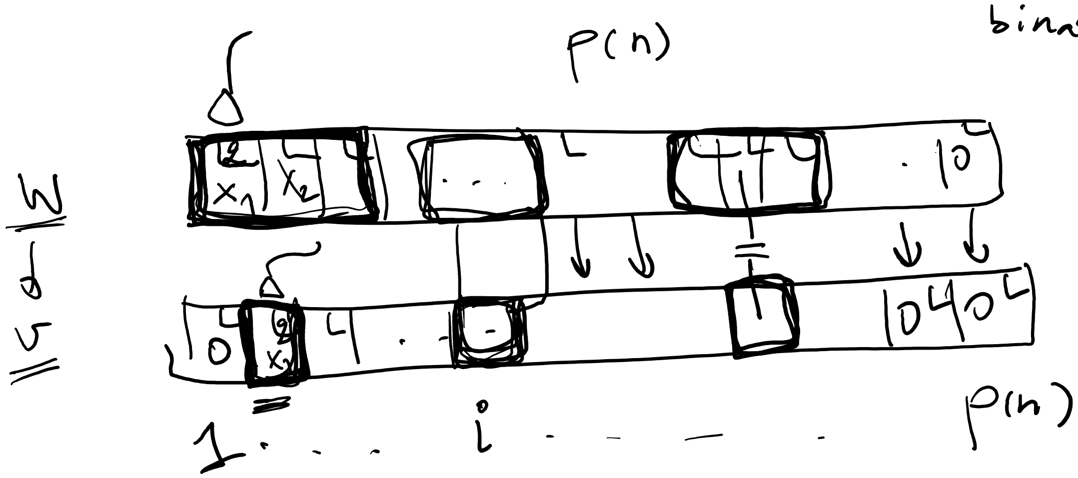
$\overline{s}, \overline{t}$ are encodings of vertices in G

$path_k(\overline{s}, \overline{t})$ ... true if $\overline{s} \xrightarrow[2^k]{} \overline{t}$

$path_0(\overline{s}, \overline{t}) = \bigwedge_{i=1}^{p(n)}$ consistency of the four cells at position $\underline{\underline{i}}$



binary encoding



$path_0(\overline{w}, \overline{v})$

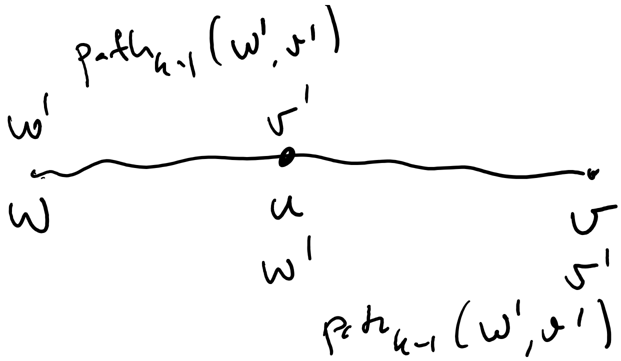
$path_0(\overline{w}, \overline{v})$ ... formula of size $O(p(n))$

$path_k(\overline{w}, \overline{v}) \equiv \exists \overline{u} \; path_{k-1}(\overline{w}, \overline{u}) \,\&\, path_{k-1}(\overline{u}, \overline{v})$.

$\times$ too big ... $2^{p(n)}$

$$path_k(\overline{w}, \overline{v}) \equiv \exists \overline{u}_k \forall \overline{w}'_k, \overline{v}'_k \left[\underbrace{\left((\overline{w}' = \overline{w}) \,\&\, (\overline{v}' = u) \right) \vee \left((\overline{w}' = \overline{u}) \,\&\, (\overline{v}' = \overline{v}) \right)}_{A} \right] \rightarrow path_{k-1}(\overline{w}', \overline{v}')$$

$path_{k-1}(w', v')$



$$\longrightarrow \underbrace{path_{k-1}(\bar{w}', \bar{v}')}_{B}$$

$$A \to B$$

$$\neg A \vee B$$

can take quantifiers to the front

$$path_k(\bar{w}, v \equiv \exists \bar{u}_k \, \forall \bar{w}'_k \, \bar{v}'_k \Big(\exists \bar{u}'_{k-1} \, \forall \bar{W}'_{k-1} \, \bar{v}'_{k-1} \big(\ldots \; path_0(\bar{w}'_1, \bar{v}'_1) \big) \Big)$$

$$path_k(\bar{w}, \bar{v}) \equiv \exists u \, \forall w'_k v'_k \big(\ldots \qquad \big)$$

$$T(n) = 2^{k(n)} \qquad k(n) \ldots \text{polynomial} \approx p(n)$$

$$\approx O\big(k(n) \cdot p(n)\big) \qquad O(p(n)) \text{ new variables}$$

$\bar{w}, \bar{v}$ — for each $path_k(\ldots)$

$path_{k(n)}(\bar{s}, \bar{t})$ is true?

$\boxed{x\,|\,00000}$ $\quad$ $\boxed{1\,|\,00000}$

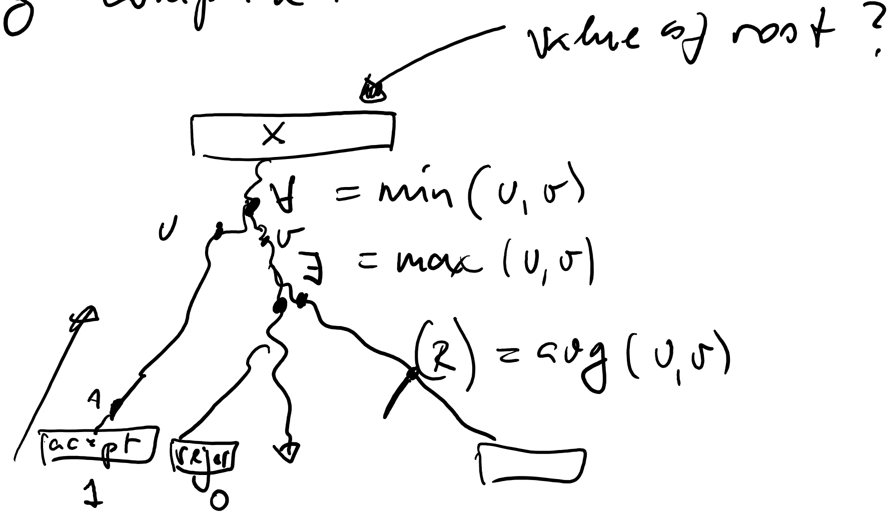
$x \in L \iff \underbrace{path_{k(n)}(\bar{s}, \bar{t})}_{\Phi}$ is true.

$x \to \Phi_x$

$\Rightarrow$ QBF are hard for PSPACE

QBF $\in$ PSPACE $\Rightarrow$ QBF is PSPACE-complete.

Alternating computation



avg R … probabilistic computation "value $> \frac{1}{2}$"

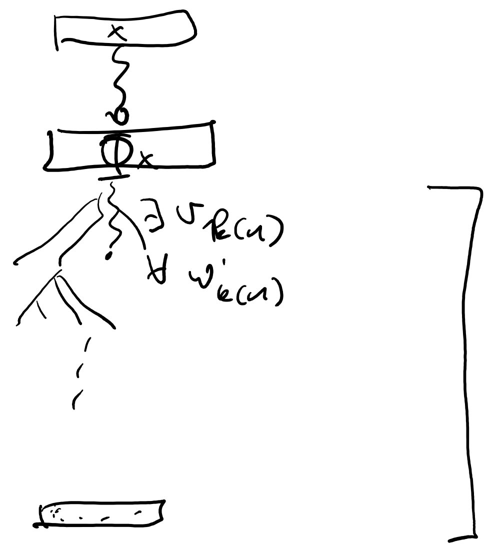
$\exists$ … nondeterministic "value $= 1$"

$\forall$ … co-nondeterministic … "value $= 1$"

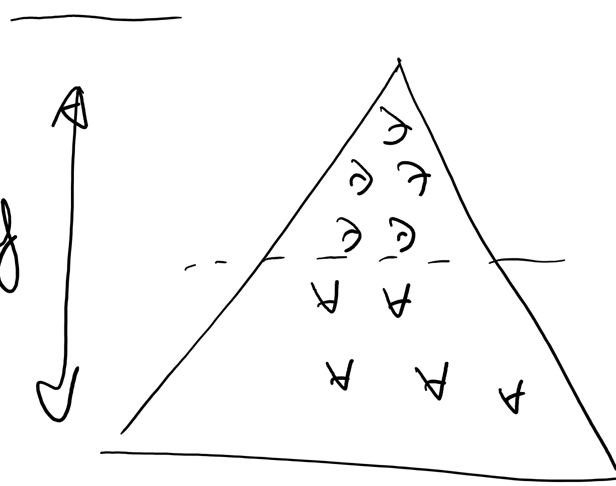
all possible R, $\exists$, $\forall$ $\in$ PSPACE

QBF is complete for PSPACE

L $\in$ PSPACE $x \longrightarrow$ QBF Φ_x ... can be decided by alt. TM in poly-time



PSPACE = Alternating poly-time computation.



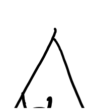
Σ_1 - computation = NP

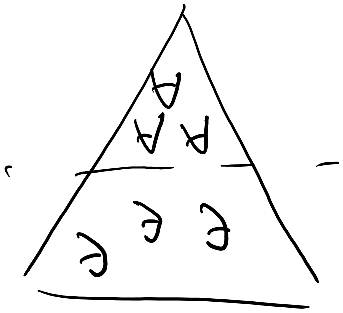
$\exists x_1 \exists x_2 \dots \exists x_n \varphi(x_1, \dots x_n)$

Σ_2 - computation

$\exists x_1 \exists x_2 \dots \exists x_\ell \forall x_{\ell+1} \dots \forall x_n \; \varphi(x_1, \dots x_n)$

Σ_2 - Formula





Σ_2 - Formula

Π_2 - computation

$\forall x_1 \forall x_2 \ldots \forall x_\ell \exists x_{\ell+1} \ldots \forall x_n$
$\varphi(x_1 \ldots x_n)$

Π_2 - formula

deciding truth of Π_2-formula is complete problem for Π_2-type alternating computation running in poly-time.

(Exc)

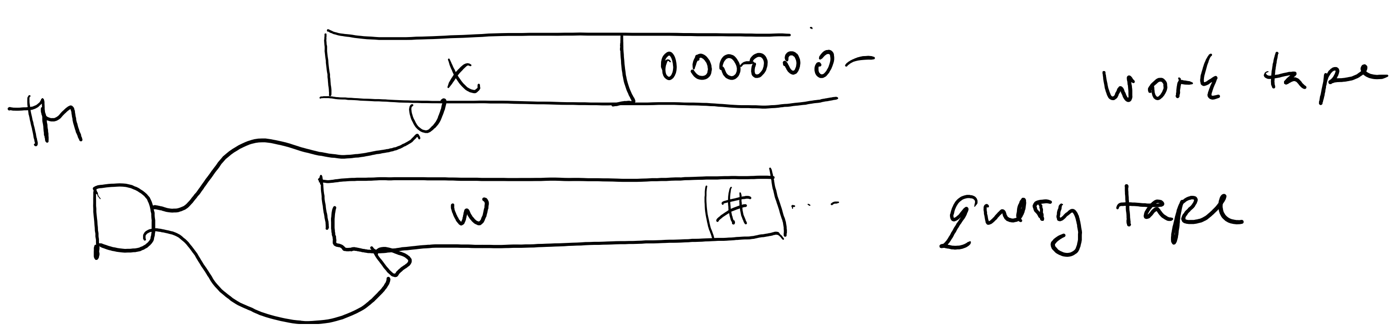
Σ_k-formulas is complete for Σ_k-computation

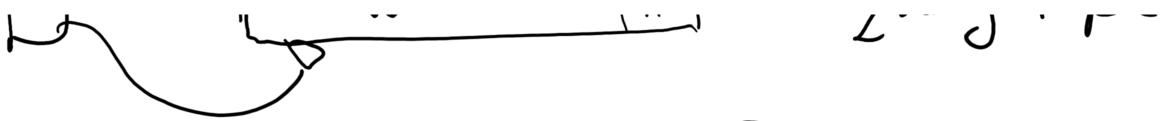
Oracle computation

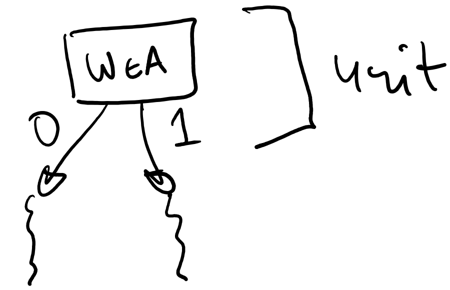
$A \subseteq \{0,1\}^*$ ... oracle

$w \in \{0,1\}^*$ "$w \overset{?}{\in} A$" .. unit time

e.g. $A = SAT$
"$\varphi \in SAT$" for unit cost







poly-time computation with oracle set A

$= P^{A}$ oracle

e.g. $NP \subseteq P^{SAT}$ $\qquad coNP \subseteq P^{SAT}$

TAUT $\in$ coNP
SAT $\in$ NP

TAUT $\in P^{SAT}$

$\forall x_1 \dots x_n \quad \underbrace{\varphi(x_1, \dots x_n)}$? $\qquad$ false $\Updownarrow$ true $\qquad \varphi \notin TAUT \Updownarrow \neg\varphi \in \underline{\underline{SAT}}$

$\exists x_1 \dots x_n \quad \neg\varphi(x_1, \dots x_n)$

$\varphi \qquad$ "$\neg\varphi \in SAT$" $\xrightarrow{YES} \varphi \notin TAUT$

$\qquad\qquad\qquad\qquad \xrightarrow{NO} \varphi \in TAUT$

$\Rightarrow TAUT \in P^{SAT} \Rightarrow coNP \subseteq P^{SAT}$

$P^{NP} \dots \bigcup_{A \in NP} P^{A} = P^{SAT}$

$P^{\dots} \dots \bigcup_{A \in NP} \dots = \dots$

$P^{coNP} = P^{NP}$

EXP^{A} $\quad PSPACE^{A}$

$L \in NP$ iff $(\{0,1\}^* \setminus L) = coL \in coNP$

$P^{NP} \in PSPACE$
$\subseteq$

$NP \subseteq PSPACE$

$P^P = P$

NP^{SAT} $\quad (NP^{NP})$

Σ_2-formulas $\in NP^{SAT}$

$\exists \bar{x}\ \forall \bar{y}\ \ \varphi(\bar{x}, \bar{y})$

$\boxed{\forall \bar{y}\, \varphi(\bar{x}, \bar{y})} \in \cancel{SAT}\ TAUT$

fixed

$NP^{NP} \equiv \Sigma_2$-computation.
$\supseteq$
"$\subseteq$"