Introduction

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- HW's, 4 sets, ≥70%
- <u>SAT</u>: Given a Boolean formula φ in CNF, decide $\exists$ an satisfying assignment for φ.

k-SAT: —"— where φ is a k-CNF

$$\varphi = C_1 \wedge C_2 \wedge C_3 \dots \wedge C_m$$

↳ clauses

$$C_i = (y_1 \vee y_2 \vee y_3 \dots \vee y_k)$$

each y_j is either one of the input variables $x_1 \dots x_n$ or its negation. = "literal"

<u>Ex</u>: $(\neg x_1 \vee x_5 \vee x_7) \wedge (x_2 \vee \neg x_3 \vee x_6) \wedge (x_1 \vee \neg x_6 \vee x_5)$

... 3-CNF

- Brute force alg for SAT: $2^n \cdot \text{poly}(m)$
- Better alg for k-SAT: $2^{n - \frac{cn}{k}} \cdot \text{poly}(m)$

$$= 2^{(1-\frac{c}{k})n} \cdot \text{poly}(m)$$

- <u>Longest Common Subsequence</u>

Ex: x = 1 0 1 1 0 1 1 1 0 0 0 1 0 1 0 1
y = 0 1 1 1 0 1 0 1 1 0 1 1 0 1 0 1

· Find the longest common subsequence of two strings x & y.

Alg.:

· dynamic programming $O(n^2)$

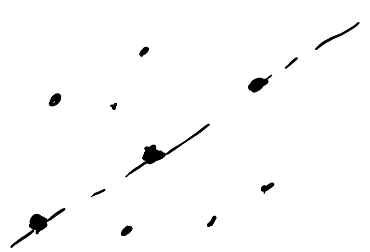
· Masek – Paterson '80 $O\left(\frac{n^2}{\lg^2 n}\right)$

no better alg. known

similar problems: edit distance

· Computational Geometry

Ex: Given n pts in plane, are any 3 of them co-linear?



$\tilde{O}(n^2)$ time

· Graph Diameter

undirected graph G

$D(G) = \max_{u,v} d(u,v)$ ... diameter of G

$R(G) = \min_{u} \max_{v} d(u,v)$ ... radius of G

$O(m \cdot n)$ ... BFS from n starting pts or All Pairs Shortest Path (APSP)

- Are there better algorithms for those problems?

- Exponential Time Hypothesis (ETH)

  3-SAT requires time $2^{\delta n}$, for some $\delta > 0$.

- Strong Exponential Time Hypothesis (SETH)

  $\forall \varepsilon > 0\ \exists k$ s.t. k-SAT cannot be solved in time $2^{(1-\varepsilon)n} \cdot \text{poly}(m)$.

- SETH $\Rightarrow$ ETH (k-SAT $\leq$ 3-SAT)

- Orthogonal Vector Problem (OVP)

  Input: $S \subseteq \{0,1\}^d$, $d \geq \lg n$, $|S| = n$

  Output: Is there $u, v \in S$ s.t. $\langle u, v \rangle = 0$?

trivial alg: $O(n^2 d)$

better alg: $O\left(n^{2 - \frac{1}{\lg \frac{d}{\lg n}}}\right)$

Thm: $O(n^{2-\varepsilon} \cdot \text{poly}(d))$ - alg. for OVP $\Rightarrow$ SETH is false.

Pf: Reduction of k-SAT to OVP

φ ... k-CNF on $x_1, \ldots x_n$

- split variables into two parts $x_1 \ldots x_{\frac{n}{2}}$, $x_{\frac{n}{2}+1} \ldots x_n$

$\varphi = C_1 \wedge C_2 \wedge C_3 \ldots C_m$ $\qquad d = m + 2$

$\forall a \in \{0,1\}^{n/2}$

$\forall\, a \in \{0,1\}^{n/2}$

$u_a \in \{0,1\}^d$ $v_a \in \{0,1\}^d$

$u_a = 0\,1\,b_1 b_2 \dots b_m$ $v_a = 1\,0\,c_1 \dots c_m$

b_i is 1 iff a doesn't make C_i true

c_i is 1 iff a doesn't make C_i true.

$\langle u_a, v_b \rangle = 0 \iff a \circ b$ is a sat. assignment to φ

$N = 2 \cdot 2^{n/2}$ ... # of vectors u_a, v_a.

$d = m+2$

$N^{2-\varepsilon} \text{poly}(d)$ alg. for OVP

$\implies 2^{(2-\varepsilon)\frac{n}{2}} \cdot \text{poly}(n^k)$ alg for k-SAT

$= 2^{n(1-\frac{\varepsilon}{2})} \cdot \text{poly}(n^k)$ ☒