

Justify every claim formally! Whenever you use a theorem, specify which one you use and explicitly verify that its assumptions are satisfied!

1. Find all local extrema of the function  $f(x, y) = xy\sqrt{1 - x^2 - y^2}$  on the set  $M := \{(x, y) : x^2 + y^2 < 1\}$ .
2. Find all local extrema of the function  $f(x, y, z) = -x^3 + 3xz + 2y - y^2 - 3z^2$ .
3. Find approximately the values of the function  $f(x, y) = x \sin y - \cos y + \cos(2y)$  at the points  $(2 + \varepsilon, \frac{\pi}{2} - \varepsilon)$  for  $|\varepsilon|$  small with error of order  $o(\varepsilon^2)$ .