Justify every claim formally! Whenever you use a theorem, specify which one you use and explicitly verify that its assumptions are satisfied!

- 1. Let $V := (\mathbb{R}^n, \|\cdot\|_1)$ be a standard *n*-dimensional vector space over reals with the ℓ_1 -norm, i.e., $\|(x_1, \ldots, x_n)\|_1 := \sum_{i=1}^n |x_i|$. Let $\|\cdot\|$ be a norm on \mathbb{R}^n . Prove that $\|\cdot\| : V \to \mathbb{R}$ is a continuous function. In other words, prove that any norm on \mathbb{R}^n is continuous with respect to the ℓ_1 -norm.
- 2. Let $f(x,y) = \sqrt{\ln(x-y)}$. Determine the domain of f and decide whether it is open or closed. Moreover, decide whether the function f is continuous and whether it attains global maximum and global minimum on its domain.
- 3. Let $g(x,y) = \frac{\sin(xy)}{\sqrt{x^2+y^2}}$. Is it possible to extend g continuously to the whole \mathbb{R}^2 ? If it is possible, specify how.