

Justify every claim formally! Whenever you use a theorem, specify which one you use and explicitly verify that its assumptions are satisfied!

1. Let $X \subset \mathbb{R}$ and $Y \subset \mathbb{R}$ be closed intervals. Let $Z := X \times Y$ and $f: Z \rightarrow \mathbb{R}$ be Riemann integrable over Z . Fix $n \in \mathbb{N}$. Prove that the set

$$\left\{ x_0 \left| (R)\overline{\int_Y} f(x_0, y) \, dy - (R)\underline{\int_Y} f(x_0, y) \, dy > \frac{1}{n} \right. \right\}$$

has measure zero.

(Hint: Use Darboux's definition.)

2. Evaluate $\iiint_V \frac{1}{(x+y+z+1)^3} dx \, dy \, dz$, where V is the region in \mathbb{R}^3 bounded by the coordinate planes and the plane $x+y+z=1$.
3. Calculate the volume of the solid in \mathbb{R}^3 bounded by the surfaces determined by the equations $y^2 = 4a^2 - 3ax$; $y^2 = ax$ and $z = \pm h$, where $a, h > 0$ are parameters.