$\begin{array}{c} \mbox{Homework 5} \\ \mbox{Deadline: } 29.3.2018 \mbox{ at } 14:00 \end{array}$

Justify every claim formally! Whenever you use a theorem, specify which one you use and explicitly verify that its assumptions are satisfied!

1. Decide whether the following integral exists as (N) and/or (R) integral and calculate its value.

$$\int_{-\pi}^{\pi} \frac{1}{(1+\cos x)^2} \sin\left(\frac{1}{1+\cos x}\right) \sin x \, \mathrm{d}x$$

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f\left(\frac{\pi}{2} - x\right) = f\left(\frac{\pi}{2} + x\right)$ holds for every $x \in \mathbb{R}$. Show that for every a > 0

$$\int_{\frac{\pi}{2}-a}^{\frac{\pi}{2}+a} f(x) \cos x \, \mathrm{d}x = 0$$

3. Calculate the following integral:

$$(R) \int_0^{\ln 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} \, \mathrm{d}x$$