Homework 4

Deadline: 22.3.2018 at 14:00

Justify every claim formally!

Note: You cannot use the Integrability criterion from the lecture in the proofs in Tasks 1 and 2, since its proof assumes validity of what you are about to prove.

1. Let $f\colon [a,b] \to \mathbb{R}$ be bounded. Let D be a partition of $[a,b]$. Prove that for every $\varepsilon > 0$ there is $\delta > 0$ such that whenever (E, C) is a partition of $[a,b]$ with points such that $\lambda(E) < \delta$, then $(s(f,D) - R(f,E,C)) < \varepsilon$.

2. Let $f\colon [a,b] \to \mathbb{R}$ be bounded. Prove that if the Riemann integral of f exists according to Riemann's definition, then it exists according to Darboux's definition and that they are equal.

3. Calculate $\displaystyle\int x \sqrt[3]{\frac{x-1}{x+2}}\,\mathrm{d}x$ on a maximal domain.