Mathematical analysis II - tutorial 8 12.4.2018

Problem 1: Let S be the region determined below. Let $f \in \mathcal{R}(S)$. Rewrite the multiple integral $(R)\int_S f$ using iterated integrals of f:

a) S is a rectangle in $\mathbb{R}^2$ with vertices $(0,0),(2,0),(2,1),(0,1)$.

b) S is a triangle with vertices $(0,0),(1,0),(1,1)$.

c) S is a trapezoid (trapezium) $(0,0),(2,0),(1,1),(0,1)$.

d) S is a parallelogram with vertices $(1,2),(2,4),(2,7),(1,5)$.

e) S is a circular sector with center at $(0,0)$ and arc endpoints at $(1,1),(-1,1)$.

f) S is an annulus determined by two circles with radii 1 and 2 both centered at $(0,0)$.

g) S is the region bounded by the line passing through $(0,2)$ and $(2,0)$ and the arc of a circle of radius 1 centered at $(0,1)$ with endpoints $(0,2)$ and $(1,1)$.

h) S is a cylinder (in $\mathbb{R}^3$) bounded by the surfaces $x^2+y^2=R^2$, $z=0$ and $z=A$, where $R,A>0$ are parameters.

i) S is a volume in $\mathbb{R}^3$ bounded by the surfaces $z=1-x^2-y^2$ and $z=0$.

j) S is the volume of the ellipsoid determined by the equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

Problem 2: Calculate the following multiple integrals:

a) $\displaystyle\iint_S \frac{1}{\sqrt{a^2-x^2-y^2}}\,\mathrm{d}x\,\mathrm{d}y$, where S is the part of a circle of radius a centered at $(0,0)$ lying in the first quadrant.

b) $\displaystyle\iint_S \sqrt{xy-y^2}\,\mathrm{d}x\,\mathrm{d}y$, where S is a triangle with vertices $(0,0),(10,1),(1,1)$.

c) $\displaystyle\iint_S \mathrm{e}^{\frac{x}{y}}\,\mathrm{d}x\,\mathrm{d}y$, where S is a curvilinear triangle bounded by the parabola $y^2=x$ and the straight lines $x=0, y=1$.

d) $\displaystyle\iint_S xy\,\mathrm{d}x\,\mathrm{d}y$, where S is the region bounded by the x-axis and the upper semi-circle $(x-2)^2+y^2=1$.

e) $\displaystyle\iiint_V x^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, where V is the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$.

f) $\displaystyle\iiint_V (x+y+z)^2\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$, where V is the common part of the paraboloid $2az \geq x^2+y^2$ and the ball $x^2+y^2+z^2 \leq 3a^2$.

Problem 3: Determine the volume of V:

a) V is a solid bounded by the xy-plane, the cylinder $x^2+y^2=ax$ and the sphere $x^2+y^2+z^2=a^2$ (interior to the cylinder), where $a>0$ is a parameter.