## Mathematical analysis II - tutorial 6

Problem 1: Calculate the area of the following shapes:
a) Circle of radius $r$
b) Area between the graph of the function $\sin ^{2} x$ on $[0, \pi]$ and the $x$-axis.
c) The bounded area determined by the following three curves: $y=\frac{1}{x}, y=\frac{1}{x^{2}}$ and $x=2$.
d) Area between the graphs of the functions $\sin x$ and $\cos x$ on $[0,2 \pi]$.

Problem 2: Calculate the lengths of the following curves:
a) Circle of radius $r$
b) Graph of the function $\frac{2}{3} x^{\frac{3}{2}}$ on $[0,3]$.
c) Graph of the function $\frac{a}{2}\left(\mathrm{e}^{\frac{x}{a}}+\mathrm{e}^{-\frac{x}{a}}\right)$ on $[0, \gamma]$, where $a, \gamma>0$ are parameters.
d) Graph of the function $\sin ^{2} x$ on $[0, \pi]$.
e) Graph of the function $t \mapsto\left(t, t^{2}, t^{3}\right)$ on $[0,1]$ (this is the so-called moment curve).

Problem 3: Calculate the volumes of the following 3-dimensional bodies:
a) A ball of radius $r$
b) A (right circular) cone with the base radius 1 and height 3 .
c) A solid of a revolution defined by the graph of a function $\sin x$ on $[0, \pi]$.

Problem 4: Calculate the surface areas of the 3-dimensional bodies given in the previous task.

Problem 5: Using a definite integral decide whether the following series converge or not:
a) $\sum_{n=1}^{\infty} \frac{1}{n}$
b) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$
c) $\sum_{n=1}^{\infty} \frac{1}{n \ln ^{2} n}$

Problem 6: Approximate the value of the following integrals up to an error $<10^{-3}$ :
a) $\int_{0}^{1} e^{-x^{2}} d x$
b) $\int_{0}^{1} x \cos \left(x^{3}\right) d x$
c) $\int_{e}^{e^{2}} \frac{1}{\ln x} \mathrm{~d} x$

