Mathematical analysis II - tutorial 6 29.3.2018

Problem 1: Calculate the area of the following shapes:

- a) Circle of radius r
- b) Area between the graph of the function $\sin^2 x$ on $[0, \pi]$ and the x-axis.
- c) The bounded area determined by the following three curves: $y = \frac{1}{x}$, $y = \frac{1}{x^2}$ and x = 2.
- d) Area between the graphs of the functions $\sin x$ and $\cos x$ on $[0, 2\pi]$.

Problem 2: Calculate the lengths of the following curves:

- a) Circle of radius r
- b) Graph of the function $\frac{2}{3}x^{\frac{3}{2}}$ on [0,3].
- c) Graph of the function $\frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$ on $[0, \gamma]$, where $a, \gamma > 0$ are parameters.
- d) Graph of the function $\sin^2 x$ on $[0, \pi]$.
- e) Graph of the function $t \mapsto (t, t^2, t^3)$ on [0, 1] (this is the so-called moment curve).

Problem 3: Calculate the volumes of the following 3-dimensional bodies:

- a) A ball of radius r
- b) A (right circular) cone with the base radius 1 and height 3.
- c) A solid of a revolution defined by the graph of a function $\sin x$ on $[0, \pi]$.

Problem 4: Calculate the surface areas of the 3-dimensional bodies given in the previous task.

Problem 5: Using a definite integral decide whether the following series converge or not:

a)
$$\sum_{n=1}^{\infty} \frac{1}{n}$$

b)
$$\sum_{n=1}^{\infty} \frac{1}{n \ln n}$$

c)
$$\sum_{n=1}^{\infty} \frac{1}{n \ln^2 n}$$

Problem 6: Approximate the value of the following integrals up to an error $< 10^{-3}$:

a)
$$\int_0^1 e^{-x^2} dx$$

b)
$$\int_0^1 x \cos(x^3) dx$$

c)
$$\int_e^{e^2} \frac{1}{\ln x} dx$$