Mathematical analysis II - tutorial 5 22.3.2018

Problem 1: Calculate the following definite integrals as (N) and (R) integrals (if they are defined)

a) $\displaystyle\int_0^1 x^\alpha \,\mathrm{d}x$

b) $\displaystyle\int_1^\infty x^\alpha \,\mathrm{d}x$

c) $\displaystyle\int_0^8 \sqrt{1+x}\,\mathrm{d}x$

d) $\int_0^1 \cos^3 x \sin x \,\mathrm{d}x$

e) $\displaystyle\int_0^5 \left|x^2-3x+2\right| \,\mathrm{d}x$

Problem 2: Calculate the following definite integrals as (N) and (R) integrals (if they are defined)

a) $\int_{1/e}^{e} |\ln x| \,\mathrm{d}x$

b) $\displaystyle\int_{-\frac{\sqrt{3}}{2}}^{0} \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$

c) $\displaystyle\int_0^{\frac{11\pi}{2}} |\cos x| \,\mathrm{d}x$

d) $\displaystyle\int_1^e x^2 \ln x \,\mathrm{d}x$

e) $\int_0^\pi x \sin x \,\mathrm{d}x$

f) $\int_0^1 \frac{1}{x^2} \sin\left(\frac{1}{x}\right) \,\mathrm{d}x.$

g) $\displaystyle\int_0^\pi \frac{1}{1+3\sin^2 x}\,\mathrm{d}x$

h) $\displaystyle\int_0^{10\pi} \left(\arctan\left(\sin^3 x + \sin(\sin x)\right) - \sin x\right)\cos x \,\mathrm{d}x$

Problem 3: Calculate the area of the following shapes:

a) Circle of radius r

b) Area between the graph of the function $\sin^2 x$ on $[0,\pi]$ and the x-axis.

c) The bounded area determined by the following three curves: $y=\frac{1}{x}$, $y=\frac{1}{x^2}$ and $x=2$.

d) Area between the graphs of the functions $\sin x$ and $\cos x$ on $[0,2\pi]$.

Problem 4: Using a definite integral decide whether the following series converge or not:

a) $\sum\limits_{n=1}^{\infty} \frac{1}{n}$

b) $\sum\limits_{n=1}^{\infty} \frac{1}{n\ln n}$

c) $\sum\limits_{n=1}^{\infty} \frac{1}{n\ln^2 n}$