Mathematical analysis II - tutorial 4 15.3.2018

Problem 1: (Standard substitutions continued) Calculate the following indefinite integrals on maximal domains:

a) $\displaystyle\int \frac{x^3}{\sqrt{x-1}}\,\mathrm{d}x$

b) $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{(x+1)^3}}\,\mathrm{d}x$

c) $\displaystyle\int \sqrt[3]{\frac{x+1}{x-1}}\,\mathrm{d}x$

d) $\displaystyle\int \frac{x^3}{\sqrt{1-x^2}}\,\mathrm{d}x$

e) $\displaystyle\int \frac{x^2+x+1}{x\sqrt{x^2-x+1}}\,\mathrm{d}x$

f) $\displaystyle\int \frac{x^2}{\sqrt{x^2-x+1}}\,\mathrm{d}x$

Problem 2: Various integrals

a) $\displaystyle\int \frac{1}{\sqrt{\mathrm{e}^x+1}}\,\mathrm{d}x$

b) $\displaystyle\int \ln^2(x+\sqrt{1+x^2})\,\mathrm{d}x$

c) $\displaystyle\int x\arccos(5x-2)\,\mathrm{d}x$

d) $\displaystyle\int \frac{1}{\mathrm{e}^{2x}+\mathrm{e}^x-2}\,\mathrm{d}x$

Problem 3: Calculate the following Riemann integrals:

a) $\displaystyle(R)\int_{-\pi}^{\pi} \operatorname{sgn} x\,\mathrm{d}x$

b) $\displaystyle(R)\int_{a}^{b} \lfloor x \rfloor\,\mathrm{d}x$

c) $\displaystyle(R)\int_{a}^{b} |x|\,\mathrm{d}x$

d) $\displaystyle(R)\int_{0}^{1} x^2\,\mathrm{d}x$

Problem 4: (Bonus homework—you can get an additional point for activity) Calculate

$$(HK)\int_0^1 \chi_{\mathbb{Q}}(x)\,\mathrm{d}x$$

where $\chi_{\mathbb{Q}}(x)$ is the characteristic function of rational numbers, i.e., it is 1 whenever $x \in \mathbb{Q}$, otherwise it is 0.