# Mathematical analysis II - tutorial 13 17.5.2018

*Problem 1:* (This problem is copied from the previous session) Find all local and global extrema of the following functions on the given set:

a) $f(x,y) = xy^2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$

b) $f(x,y) = \sqrt{3}x - y + 2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^2 + 2x + y^2 \leq 0\}$

c) $f(x,y) = x + 2y + \frac{3}{4}x^2 + xy + 2y^2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : y^2 - 2 \leq x \leq -y^2 + 2\}$

*Problem 2:* Find all local and global extrema of the following functions on the given set:

a) $f(x,y) = y$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^3 + y^3 - 3xy = 0\}$

b) $f(x,y) = y^2 - 2x + x^2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x = y^2\}$

c) $f(x,y) = x^2 + y^2 - 6x - 4y + 11$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 - 4x \leq 5\}$

d) $f(x,y) = x^2 - y^2$ on the set $M = \left\{(x,y) \in \mathbb{R}^2 : y + \mathrm{e}^{-x^2} = 1\right\}$

e) $f(x,y) = x^2 - 4xy + y^2 + 4y$ on the set $M = \{(x,y) \in \mathbb{R}^2 : 0 \leq x \leq 1, 0 \leq y \leq 1\}$

f) $f(x,y,z) = xyz$ on the set $M = \{(x,y,z) \in \mathbb{R}^3 : x + y + z = 5, xy + yz + zx = 8\}$

*Problem 3:* Let the function $y = y(x)$ be determined implicitly by the equation $F(x,y) = 0$. Calculate the derivative of $y$ at the given point. Calculate also its second-order derivatives:

a) $F(x,y) = x^2 + y^2 - 4x - 10y + 4$ at $(x_0, y_0) = (6, 2)$.

b) $F(x,y) = x \sin y - \cos y + \cos 2y$ at $(x_0, y_0) = (1, 0)$.

c) $F(x,y) = x^2 - xy + 2y^2 + x - y - 1$ at $(x_0, y_0) = (0, 1)$.

*Problem 4:* Let the function $z = z(x,y)$ be determined implicitly by the equation $F(x,y,z) = 0$. Calculate the derivative of $z$ at the given point (you can also calculate its second-order derivatives if you want):

a) $F(x,y,z) = \cos^2 x + \cos^2 y + \cos^2 z - 1$ at $(x_0, y_0, z_0) = (\frac{\pi}{3}, \frac{\pi}{2}, \frac{\pi}{6})$.

b) $F(x,y,z) = \mathrm{e}^z + x^2 y + z + 5$ at $(x_0, y_0, z_0) = (1, -6, 0)$.

*Problem 5:* Express the given set $M$ locally on a neighbourhood of the given point as a graph of a function and determine the tangent line to that function at the given point.

a) $M = \{(x,y) \in \mathbb{R}^2 : (x^2 + y^2)^2 - 2(x^2 - y^2) = 0\}$ around the point $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

b) $M = \left\{(x,y) \in \mathbb{R}^2 : \ln\left(\sqrt{x^2 + y^2}\right) = \arctan \frac{y}{x}\right\}$ around the point $(1, 0)$

*Problem 6:* Let the function $z = z(x,y)$ be represented implicitly as

$$x^2 + y^2 + z^2 - 2x + 4y - 6z = 11.$$

Find all extrema of the function $z$.