Mathematical analysis II - tutorial 12 10.5.2018

Problem 1: Find all partial derivatives and the derivative of the following functions using the rule for the derivative of a composed mapping.

a) $f(t) = e^{x(t)-2y(t)}$, where $x(t) = \sin t$, $y(t) = t^3$ b) $f(t) = \arcsin(x-y)$, where x = 3t, $y = 4t^3$ (from now on we write just x for x(t), and similarly for y)

c)
$$f(r,\varphi) = x^2y - xy^2$$
, where $x = r\cos\varphi$ and $y = r\sin\varphi$

Problem 2: Let $f \in \mathcal{C}^2(\mathbb{R}^3)$ be a real-valued function. Express all second-order partial derivatives of the function p(x, y, z) = f(x, xy, xyz) using the partial derivatives of f.

Problem 3: Find all local extrema of the following functions:

a)
$$f(x, y) = x^2 + |\arctan y| - x^8$$

b) $f(x, y) = (|x| + |y|)^2 - (|x| + |y|)^4$
c) $f(x, y) = xy \ln(x^2 + y^2)$
d) $f(x, y) = x(3 - x^2) - y^2$
e) $f(x, y) = \frac{x^3}{3} - xy + \frac{y^2}{2}$
f) $f(x, y, z) = x + \frac{y^2}{4x} + \frac{z^2}{y} + \frac{2}{z}$

Problem 4: Find all local and global extrema of the following functions on the given set:

a) $f(x,y) = xy^2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ b) $f(x,y) = \sqrt{3}x - y + 2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : x^2 + 2x + y^2 \le 0\}$ c) $f(x,y) = x + 2y + \frac{3}{4}x^2 + xy + 2y^2$ on the set $M = \{(x,y) \in \mathbb{R}^2 : y^2 - 2 \le x \le -y^2 + 2\}$