## Mathematical analysis II - tutorial 12

Problem 1: Find all partial derivatives and the derivative of the following functions using the rule for the derivative of a composed mapping.
a) $f(t)=\mathrm{e}^{x(t)-2 y(t)}$, where $x(t)=\sin t, y(t)=t^{3}$
b) $f(t)=\arcsin (x-y)$, where $x=3 t, y=4 t^{3}$ (from now on we write just $x$ for $x(t)$, and similarly for $y$ )
c) $f(r, \varphi)=x^{2} y-x y^{2}$, where $x=r \cos \varphi$ and $y=r \sin \varphi$

Problem 2: Let $f \in \mathcal{C}^{2}\left(\mathbb{R}^{3}\right)$ be a real-valued function. Express all second-order partial derivatives of the function $p(x, y, z)=f(x, x y, x y z)$ using the partial derivatives of $f$.

Problem 3: Find all local extrema of the following functions:
a) $f(x, y)=x^{2}+|\arctan y|-x^{8}$
b) $f(x, y)=(|x|+|y|)^{2}-(|x|+|y|)^{4}$
c) $f(x, y)=x y \ln \left(x^{2}+y^{2}\right)$
d) $f(x, y)=x\left(3-x^{2}\right)-y^{2}$
e) $f(x, y)=\frac{x^{3}}{3}-x y+\frac{y^{2}}{2}$
f) $f(x, y, z)=x+\frac{y^{2}}{4 x}+\frac{z^{2}}{y}+\frac{2}{z}$

Problem 4: Find all local and global extrema of the following functions on the given set:
a) $f(x, y)=x y^{2}$ on the set $M=\left\{(x, y) \in \mathbb{R}^{2}: x^{2}+y^{2}=1\right\}$
b) $f(x, y)=\sqrt{3} x-y+2$ on the set $M=\left\{(x, y) \in \mathbb{R}^{2}: x^{2}+2 x+y^{2} \leq 0\right\}$
c) $f(x, y)=x+2 y+\frac{3}{4} x^{2}+x y+2 y^{2}$ on the set $M=\left\{(x, y) \in \mathbb{R}^{2}: y^{2}-2 \leq x \leq-y^{2}+2\right\}$

