Mathematical analysis II - tutorial 11 3.5.2018

Problem 1: Calculate all (first-order) partial derivatives of the following functions:

a) $x^2 + 4xy^3 + y^5$

b) x^{y^2}

c) $(1+x)^k(1+y)^l(1+z)^m$, where k, l, m are parameters.

d) $\ln(1+x)\ln(1+y)$

e) $(1+x)^{(1+y)}$

Problem 2: Calculate the mixed second-order partial derivatives of the following functions and check that they are equal:

a) $f(x,y) = x^3 + 4xy - y^2$

b) $f(x,y) = x^{y^2}$

c)

$$f(x,y) = \begin{cases} xy & \text{for } |x| \geq |y| \\ 0 & \text{otherwise} \end{cases}$$

at the point $(0,0)$

Problem 3: Show that the following functions are differentiable at the point $(x,y) = (0,0)$ and write down the matrix of the derivative.

a) $(1+x)^k(1+y)^l$, where $k, l \in \mathbb{N}$ are parameters.

b) $\ln(1+x)\ln(1+y)$

c) $(1+x)^{1+y}$

Problem 4: Calculate the derivative of the following functions wherever it exists. Determine also a tangent hyperplane at the given point.

a) $f(x,y) = -x^2 - y^2 + 2x + 4y - 4$, tangent hyperplane at $a = (1,1,0)$

b) $f(x,y,z) = \left(\frac{x}{y}\right)^z$, tangent hyperplane at $a = (\mathrm{e}, 1, 2, \mathrm{e}^2)$

c) $f(x,y) = \arctan\frac{x-y}{x+y}$, tangent hyperplane at $a = (1,1,f(1,1))$

Problem 5: Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \sqrt{|x||y|}.$$

Calculate the (first-order) partial derivatives of f at the point $(0,0)$. Decide whether the function f is differentiable at $(0,0)$ or not.

Problem 6: Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by

$$f(x,y) = \sqrt{x^2 + 4y^2} + 1.$$

Calculate the (first-order) partial derivatives of f at the point $(0,0)$. Decide whether the function f is differentiable at $(0,0)$ or not.

Problem 7: Consider the function $f = \frac{x^3 y}{x^2+y^2}$. Extend f to a function defined on the whole $\mathbb{R}^2$ so that the extension is differentiable everywhere, if possible. Determine the derivative.