Mathematical analysis II - tutorial 10 26.4.2018

Problem 1: Calculate the following limits or prove that they do not exist.

a) $\lim\limits_{(x,y)\to(0,2)} \dfrac{\sin xy}{x}$

b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\sin(x^2+y^2)}{x^2+y^2}$

c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2-y^2}{x^2+y^2}$

d) $\lim\limits_{(x,y)\to(0,0)} \dfrac{\ln(1+xy)}{|x|+|y|}$

Problem 2: Determine the domain of the following functions. Is it an open or a closed set? Is it bounded? Are the functions continuous? Are the functions bounded?

a) $\frac{2xy}{x^2+y^2}$

b) $\cos\left(\frac{1}{xy}\right)$

c) $\frac{1}{1-x^2-y^2}$

d) $\ln\sqrt{x^2+y^2}$

e) $\frac{1}{(x-y)^2}$

f) $\frac{\sin xy}{|x|+|y|}$

Problem 3: Determine the domain of the following functions. Is it an open or a closed set? Determine whether the given function is continuous and/or bounded.

a) $f(x,y,z) = \sqrt{\frac{z^2}{x^2+y^2}-1}$

b) $f(x,y) = \arcsin(xy)$

c) $f(x,y) = \frac{x^2y}{x^2+y^2}$ for $(x,y \neq (0,0))$; moreover, we set $f(0,0)=0$.

d) $f(x,y) = \frac{2x^2y}{x^4+y^2}$ for $(x,y \neq (0,0))$; moreover, we set $f(0,0)=0$.