Mathematical analysis II - tutorial 1 22.2.2018

Problem 1: Fill in the table:

$f(x)$	$\int f(x)\,\mathrm{d}x$	domain
$x^a,$ where $a \in \mathbb{R} \setminus \{-1\}$		
$x^a,$ where $a \in \mathbb{Z}, a < -1$		
$x^a,$ where $a \in \mathbb{Z}, a > -1$		
$\frac{1}{x}$		
e^x		
a^x, where $a > 0$		
$\sin x$		
$\cos x$		
$\frac{1}{\cos^2 x}$		
$\frac{1}{1+x^2}$		
$\frac{1}{\sqrt{1-x^2}}$		

Problem 2: Calculate the following indefinite integrals and determine the interval(s) on which your result holds true:

a) $\int x^3 + 2x^2 + \frac{x}{17}\,\mathrm{d}x$

b) $\int 18e^x + 16e^{8x} + \frac{1}{x} - 3\cos x\,\mathrm{d}x$

c) $\int \sqrt{x^6}\,\mathrm{d}x$

d) $\int \frac{(1-x)^3}{x\sqrt[3]{x}}\,\mathrm{d}x$

e) $\int \tan^2 x\,\mathrm{d}x$

Problem 3: Using integration by parts calculate the following:

a) $\int x \sin x\,\mathrm{d}x$

b) $\int x^a \ln x\,\mathrm{d}x$, where $a > 0$

c) $\int \frac{x^2}{e^x}\,\mathrm{d}x$

d) $\int e^x \sin x\,\mathrm{d}x$

e) $\int \ln x\,\mathrm{d}x$

f) $\int \arcsin x\,\mathrm{d}x$

g) $\int \operatorname{arctg} x\,\mathrm{d}x$

h) $\int \sqrt{1-x^2}\,\mathrm{d}x$

Problem 4: Find recurrent formulas for the following expressions. Do not forget to determine the domain on which is your formula valid.

a) $\int \sin^n x \, \mathrm{d}x$

b) $\int \cos^n x \, \mathrm{d}x$

c) $\int e^x x^n \, \mathrm{d}x$

Problem 5: Assuming that $\int \frac{1}{\ln x} \, \mathrm{d}x$ is not expressible in a closed form on any interval, prove that $\int \ln \ln x \, \mathrm{d}x$ is not expressible in a closed form on any interval.