THE TREE EVALUATION PROBLEM

CONTEXT & RECENT RESULTS

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COLUMBIA, 2023.12.01

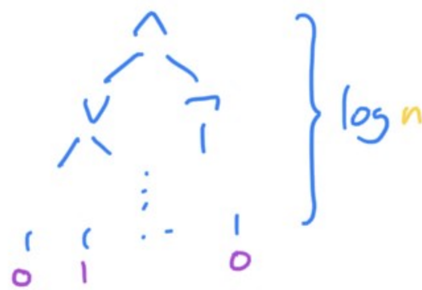
Tree Evaluation

$$NC^1 \subseteq L \subseteq NL \subseteq NC^2 \subseteq P$$

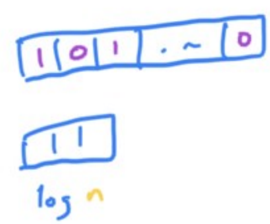
TREE EVALUATION

$$NC^1 \subseteq L \subseteq NL \subseteq NC^2 \subseteq P$$

NC^1

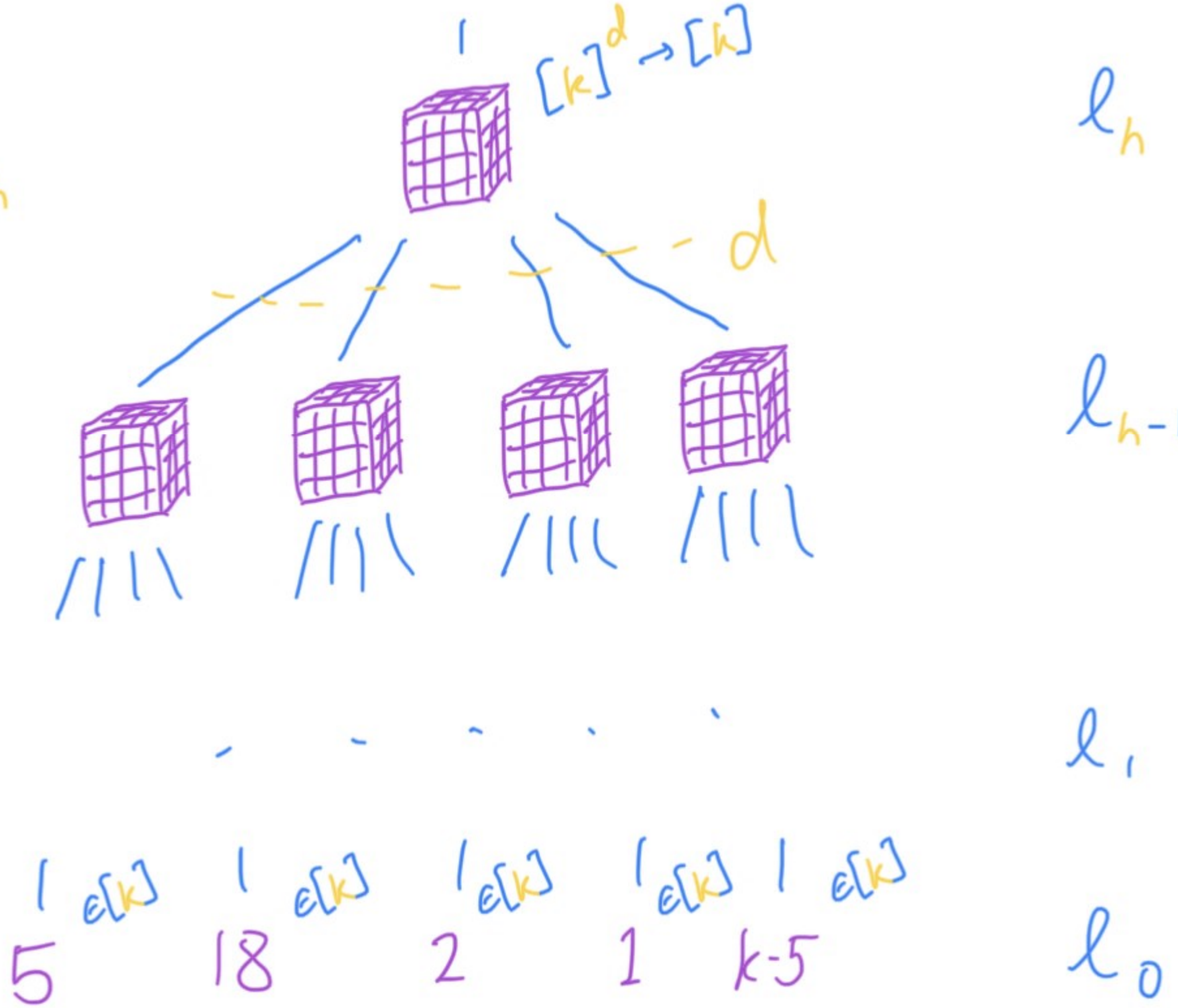


L

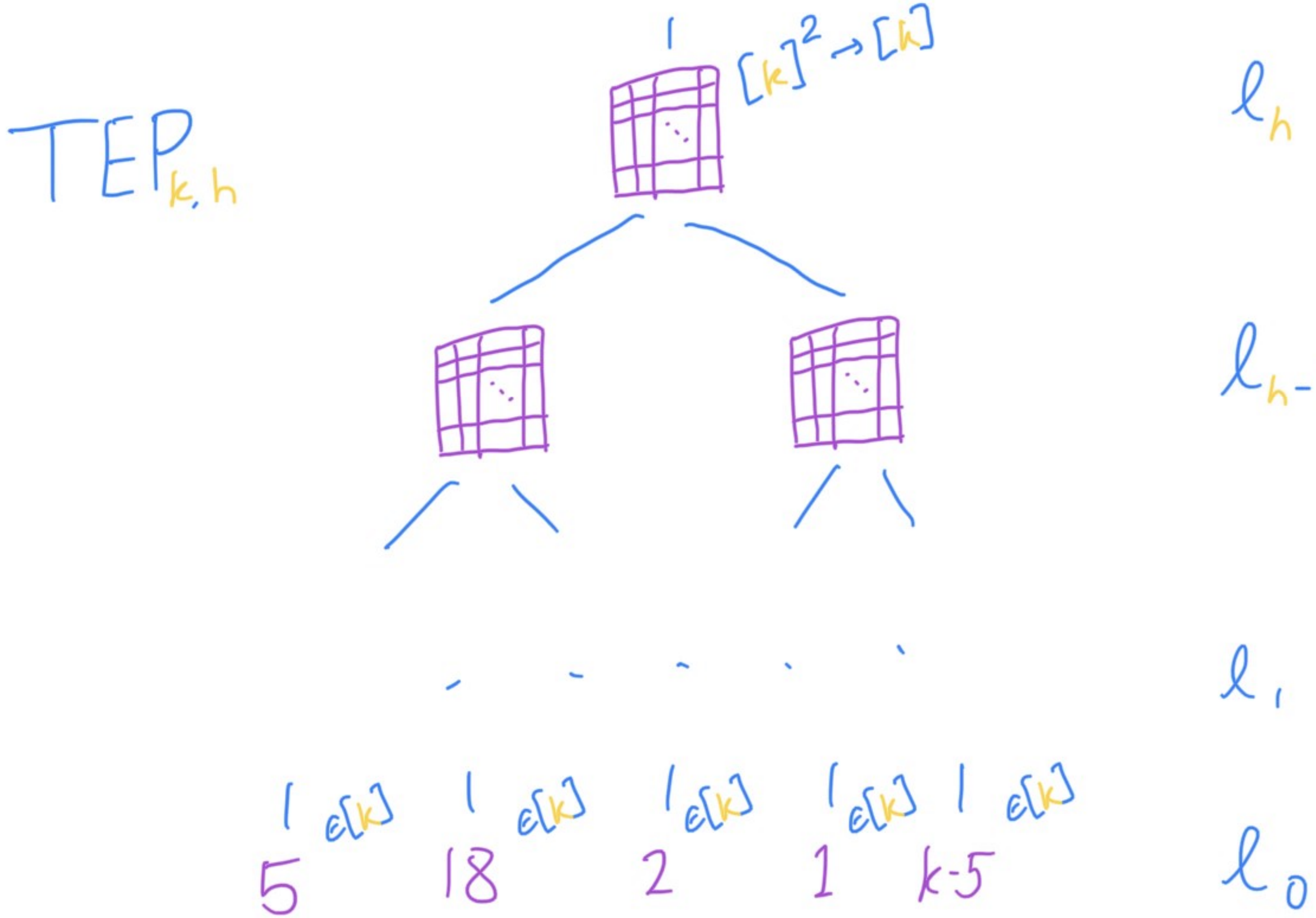


Tree Evaluation

$TEP_{k,d,h}$

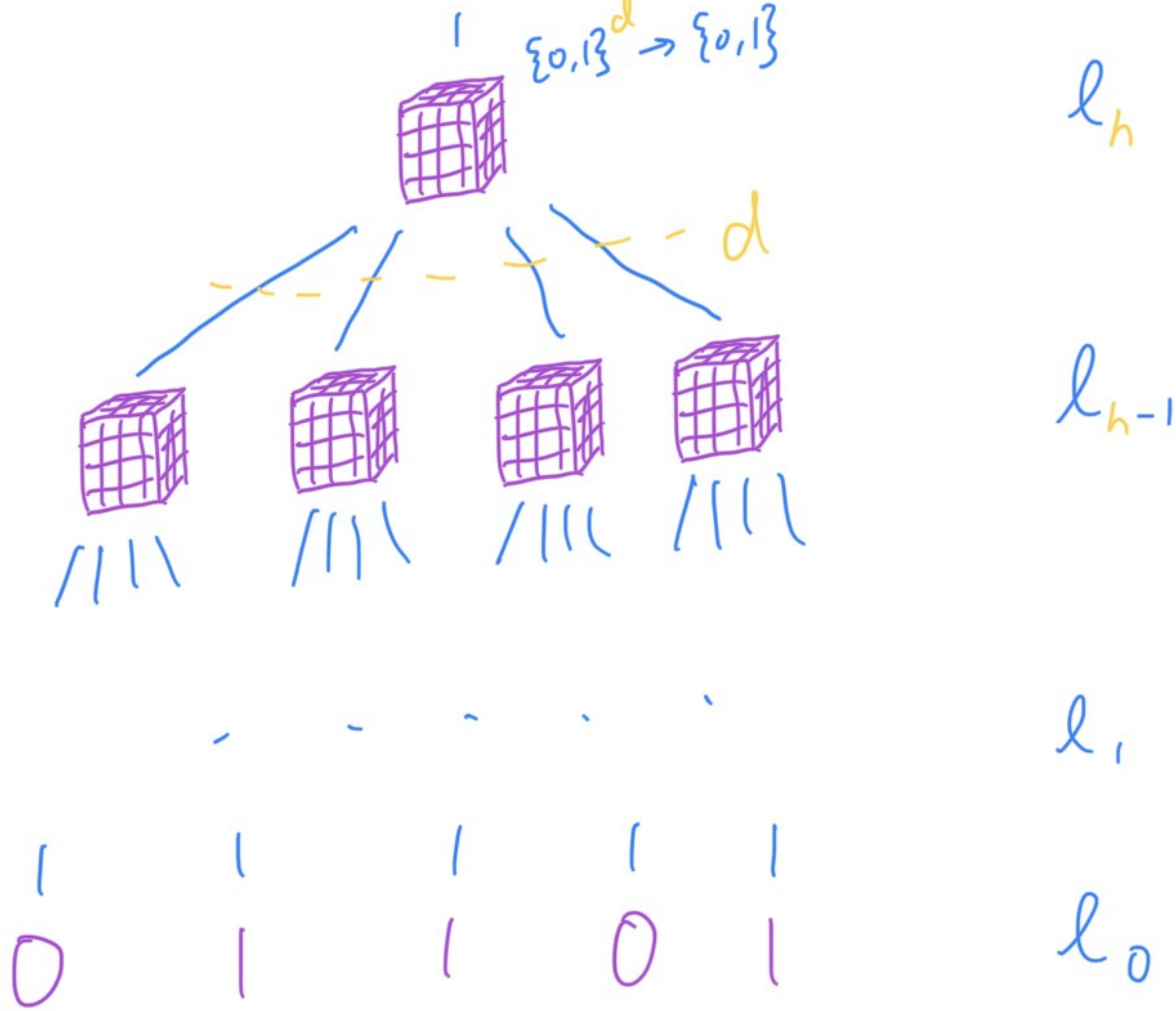


TREE EVALUATION



TREE EVALUATION

$TEP_{d,h}$
(IMX)



Tree Evaluation

$$\mathrm{TEP}_{k,d,h} \in P$$

(or even NC^2)

Tree Evaluation

$$\mathrm{TEP}_{k,d,h} \in \mathrm{P}$$

(or even NC^2)

$$NC^1 \leq \mathrm{TEP}_{2,2,\log n}$$

Tree Evaluation

$$\leq \quad \mathrm{TEP} \quad \in$$

$$NC^1 \subseteq L \subseteq NL \subseteq NC^2 \subseteq P$$

Hardness of TEP

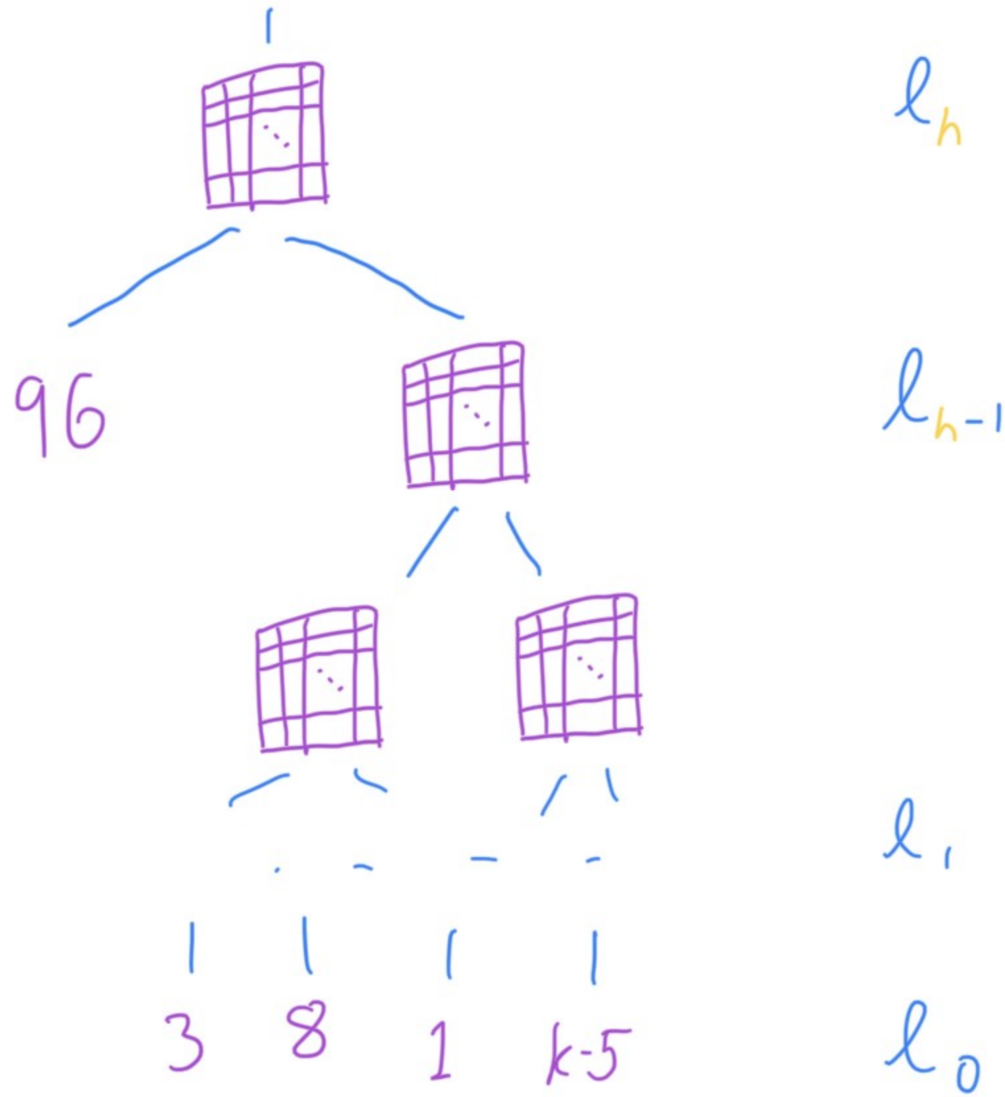
Conjecture [KRW'95]: $TEP_{d,h} \notin NC^1$

Hardness of TEP

<u>Conjecture</u> [KRW'95]: $TEP_{d,h} \notin NC^1$

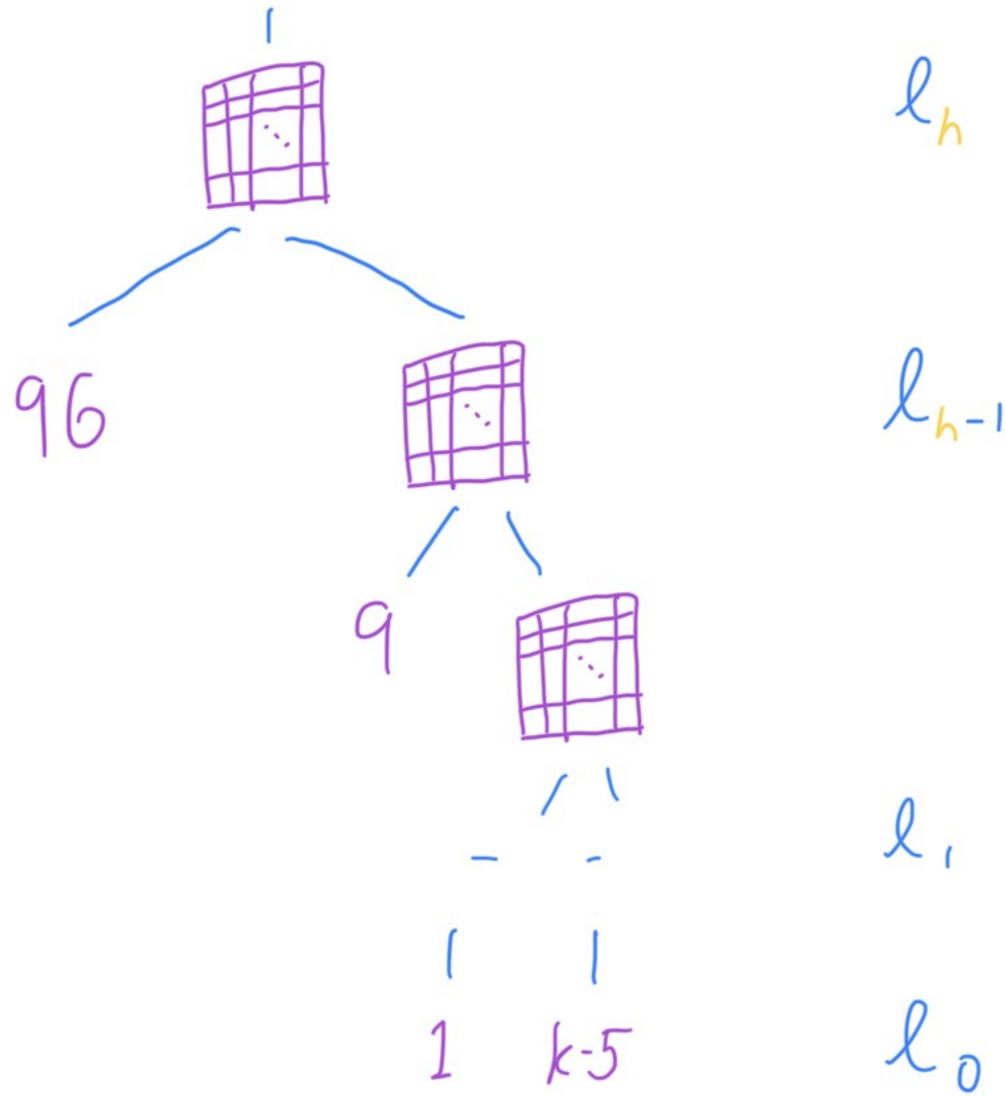
<u>Conjecture</u> [CMWBS'12]: $TEP_{k,h} \notin L$

HARDNESS OF TEP



space = log k

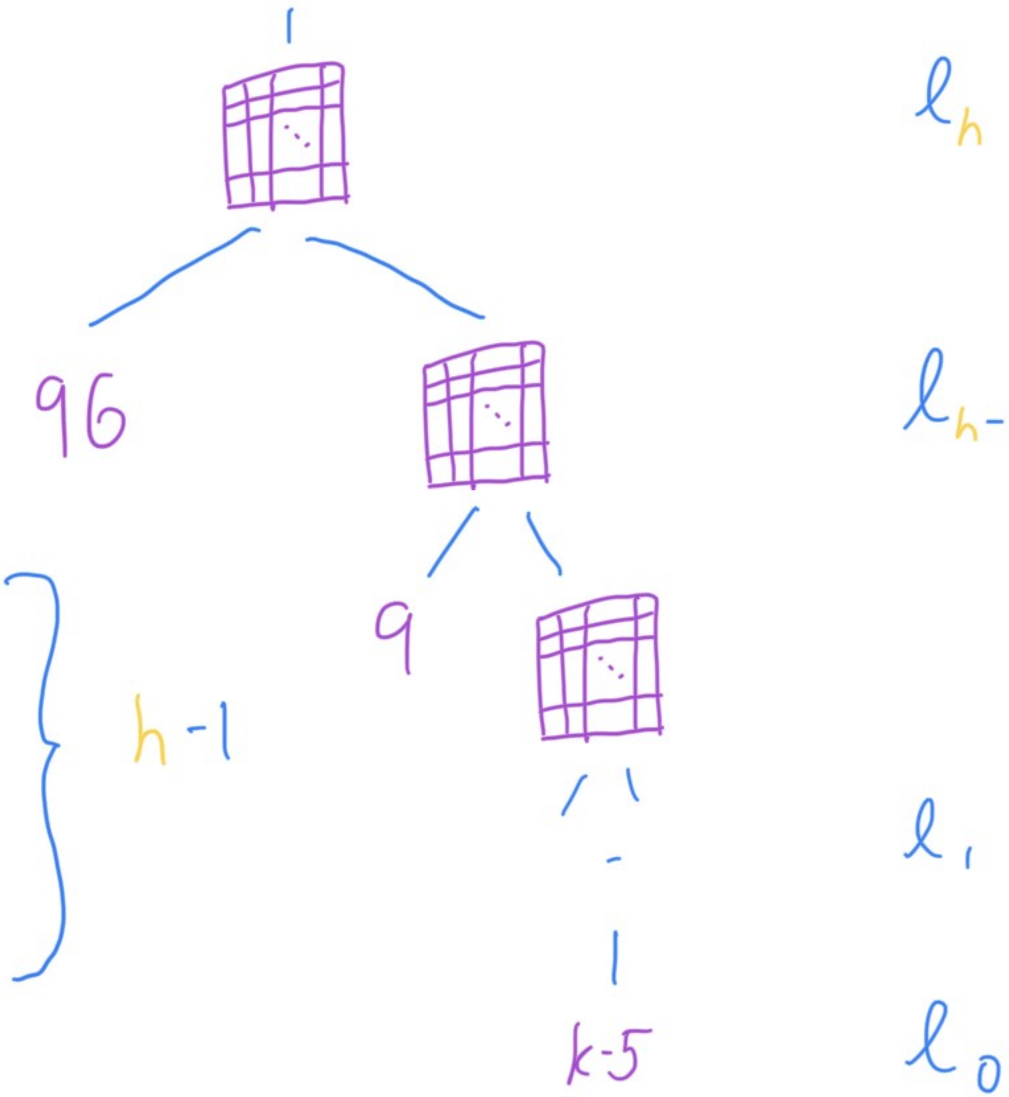
Hardness of TEP



space = $\log k$

$+ \log k$

Hardness of TEP



$$\text{space} = \left.\begin{aligned} &\log k \\ &+ \log k \\ &+ \cdots \\ &+ \log k \end{aligned}\right\} h-1$$

HARDNESS OF TEP

<u>CONJECTURE</u> [CMWBS'12]: $\text{TEP}_{k,h}$ requires space $\Omega(h \log k) = \Omega(\log^2 n)$

$(|\text{TEP}_{k,h}| = 2^h \operatorname{poly} k)$

HARDNESS OF TEP

CONJECTURE [CMWBS'12]: $TEP_{k,h}$ requires space $\Omega(h \log k) = \Omega(\log^2 n)$

thrifty ✓

read-once ✓

$(|TEP_{k,h}| = 2^h \operatorname{poly} k)$

Hardness of TEP

Conjecture [CMWBS'12]: $TEP_{k,h}$ requires

non-deterministic space $\Omega(h \log k) = \Omega(\log^2 n)$

thrifty ✓

read-once ✓

$(|TEP_{k,h}| = 2^h \operatorname{poly} k)$

Hardness of TEP

<u>Conjecture</u> [KRW'95]: $TEP_{d,h}$ requires depth $\Omega(dh) = \Omega(\log^2 n / \log\log n)$

$(|TEP_{d,h}| = d^h \, 2^d)$

EASINESS OF TEP?

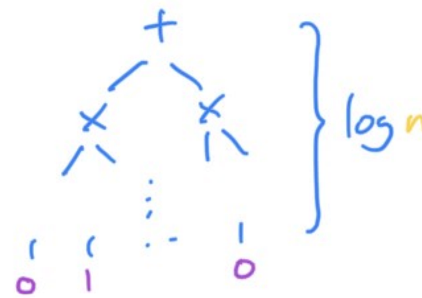
<u>BARRINGTON'S THEOREM</u>: NC^1 can be computed with permutation BPs of $\mathrm{poly}(n)$ length and width 5.

Easiness of TEP?

Theorem [BC'89]: $\#NC^1 \subseteq L$.

Easiness of TEP?

$\#NC^1$

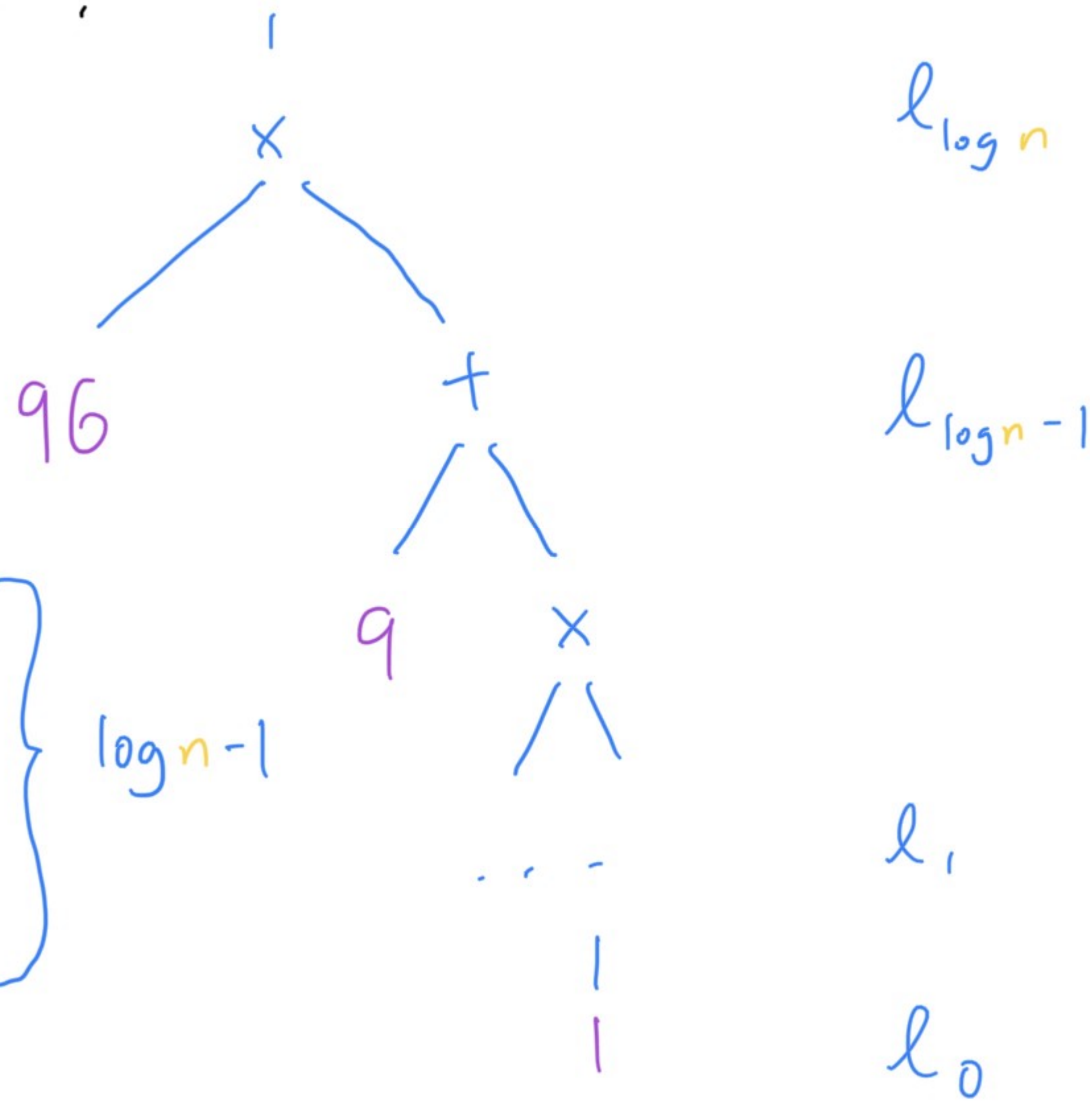


$$\#NC^1$$

$$\subseteq \qquad \subseteq$$

$$NC^1 \subseteq L \subseteq NL \subseteq NC^2 \subseteq P$$

Easiness of TEP?



$$\text{space} = \underbrace{\log n + \log n + \cdots + \log n}_{\log n - 1}$$

Ben-Or & Cleve

Proof [BC'89]:

two uses of space:

1) storage

2) computation

BEN-OR & CLEVE

PROOF [BC'89]:

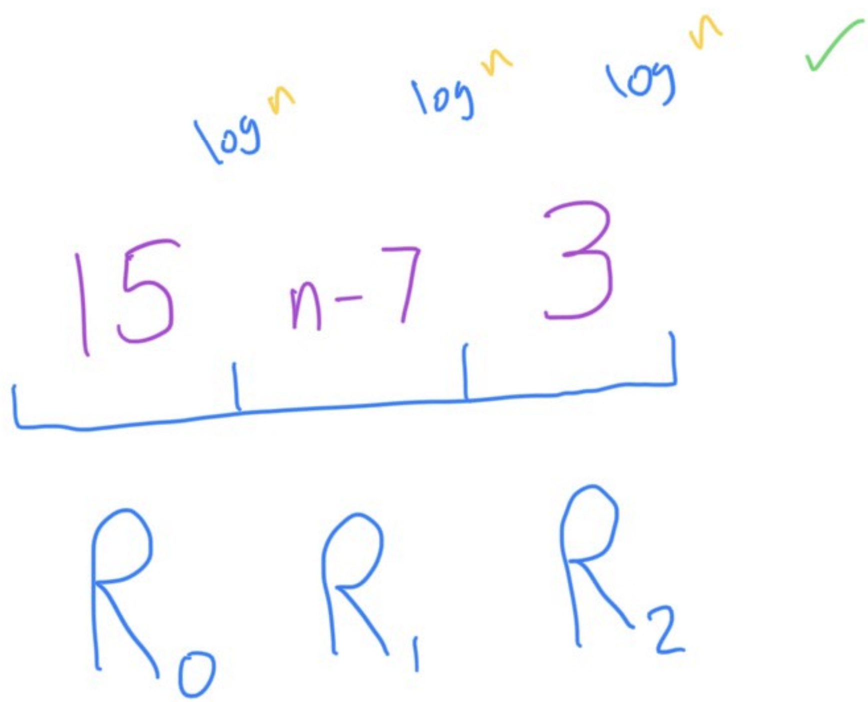
two uses of space:

1) storage

2) computation

BOTH AT ONCE?

Ben-Or & Cleve



Ben-Or & Cleve

$15 \quad n-7 \quad 3$ ← storage (old)

$R_0 \quad R_1 \quad R_2$

Ben-Or & Cleve

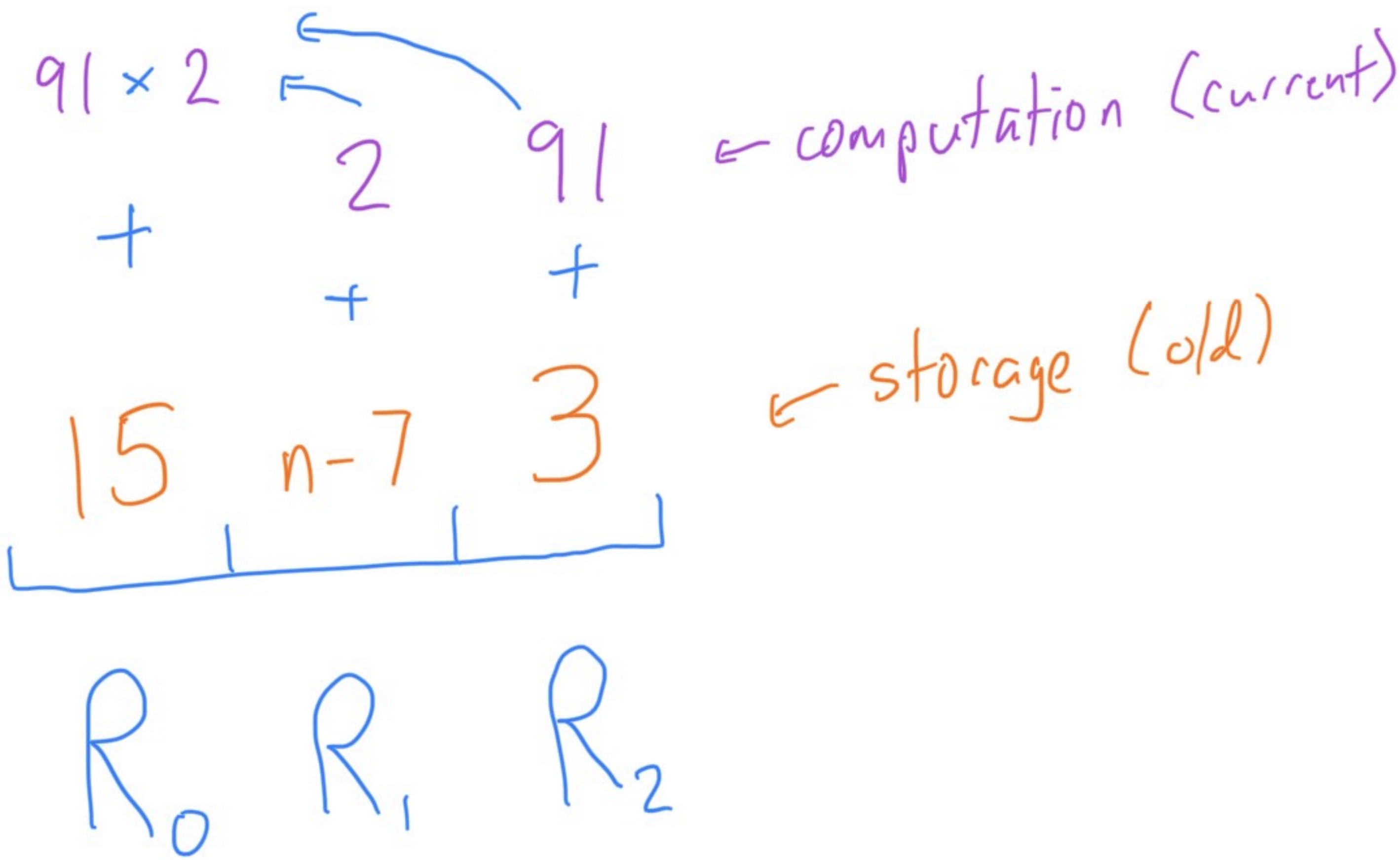
91 ← computation (current)

$+$

$15 \quad n-7 \quad 3$ ← storage (old)

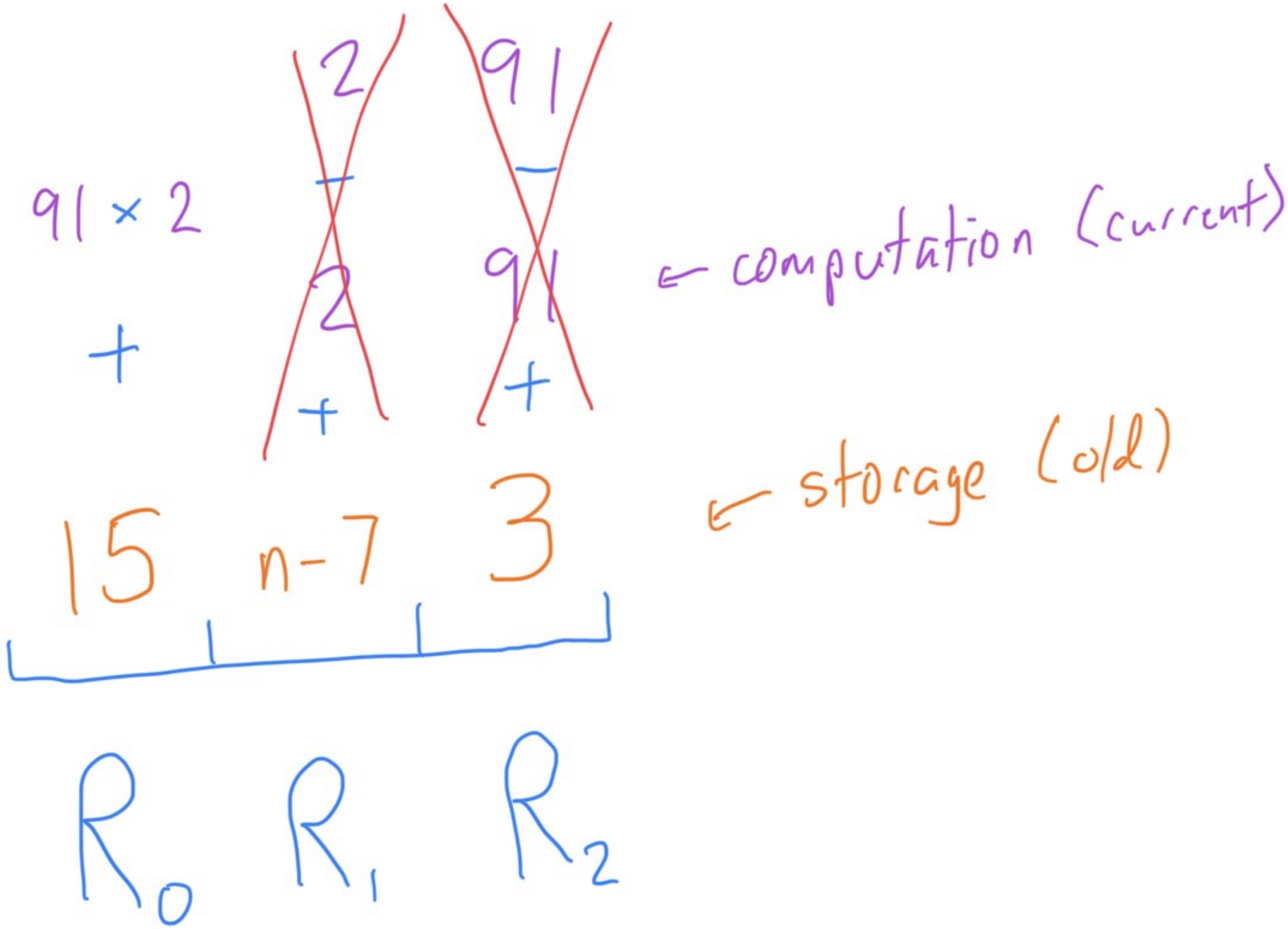
$R_0 \quad R_1 \quad R_2$

Ben-Or & Cleve



91×2
2
91
← computation (current)
+
+
+
15
n-7
3
← storage (old)
R_0
R_1
R_2

Ben-Or & Cleve

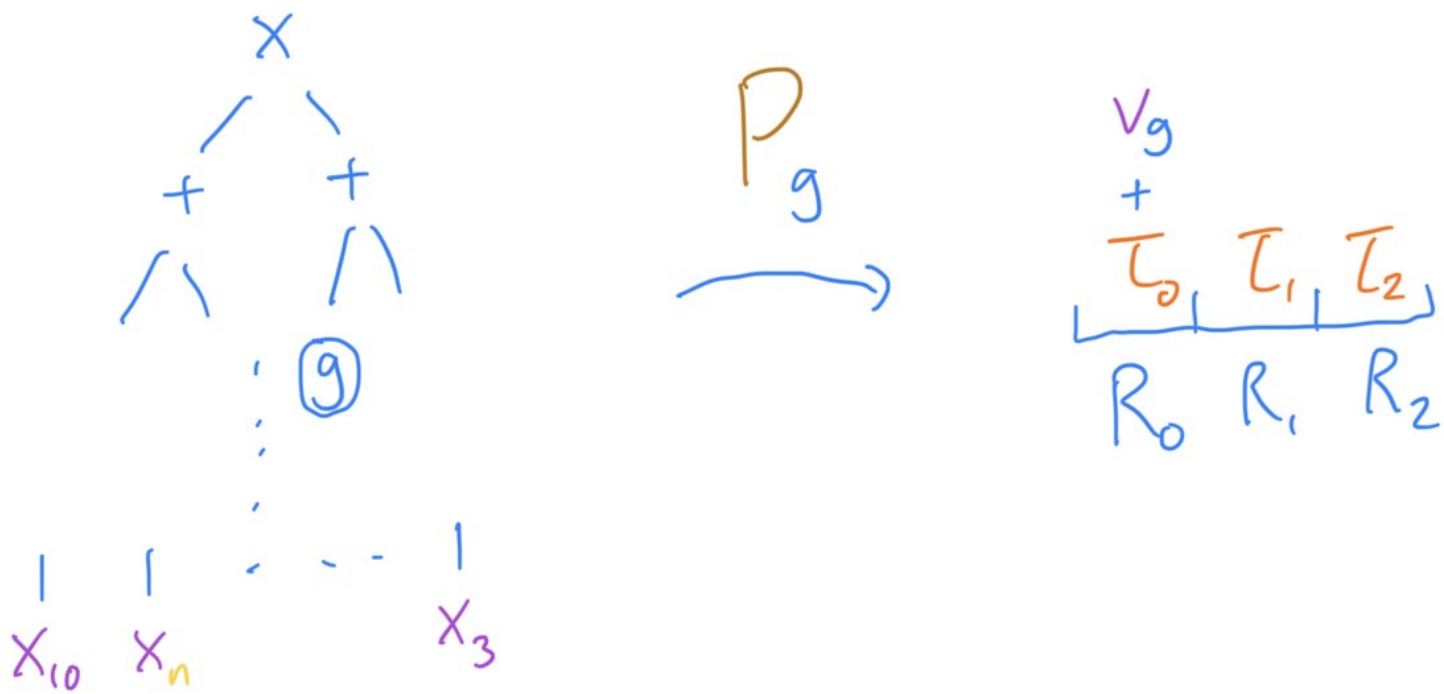


Ben-Or & Cleve

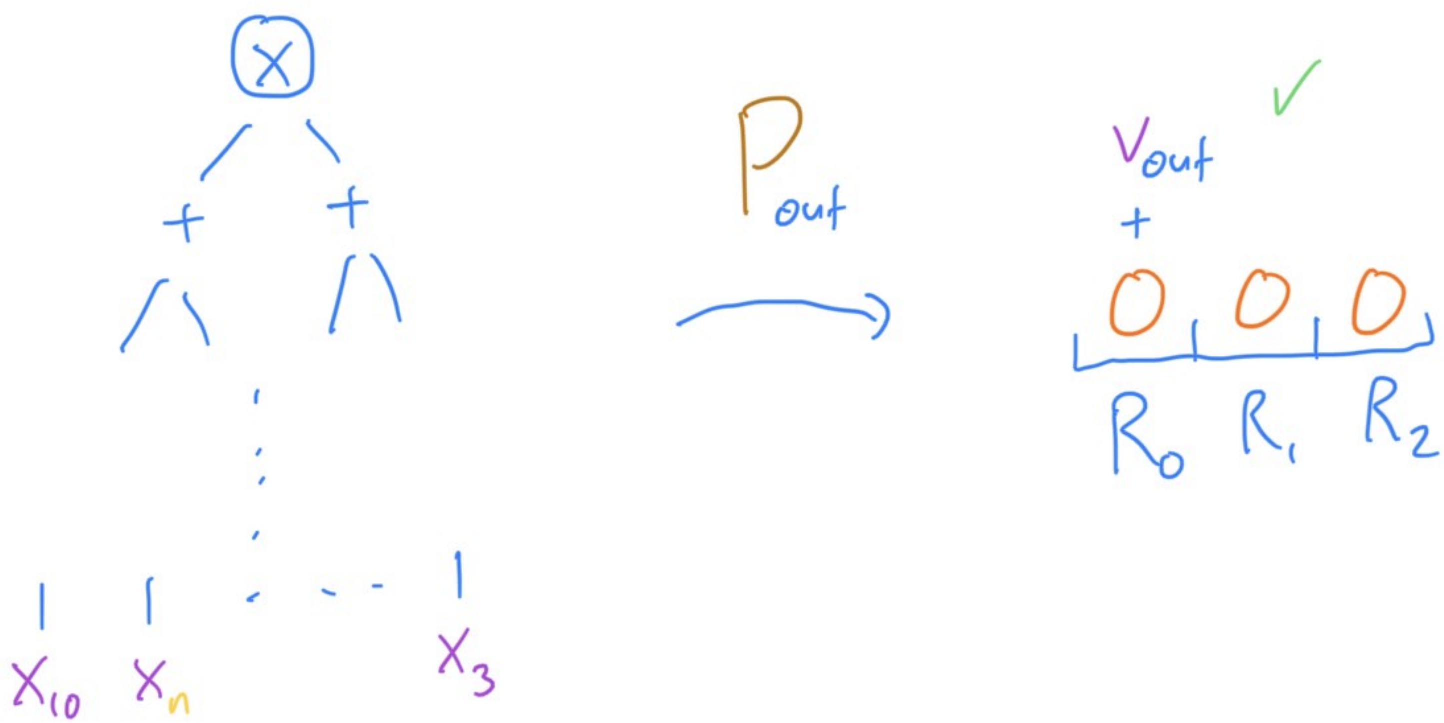
<u>Lemma</u>: $\forall g \in \mathcal{C}$, $\exists P_g$ s.t.

$$P_g: \quad \begin{aligned} R_0 &\leftarrow R_0 + v_g \\ R_1 &\leftarrow R_1 \\ R_2 &\leftarrow R_2 \end{aligned}$$

Ben-Or & Cleve

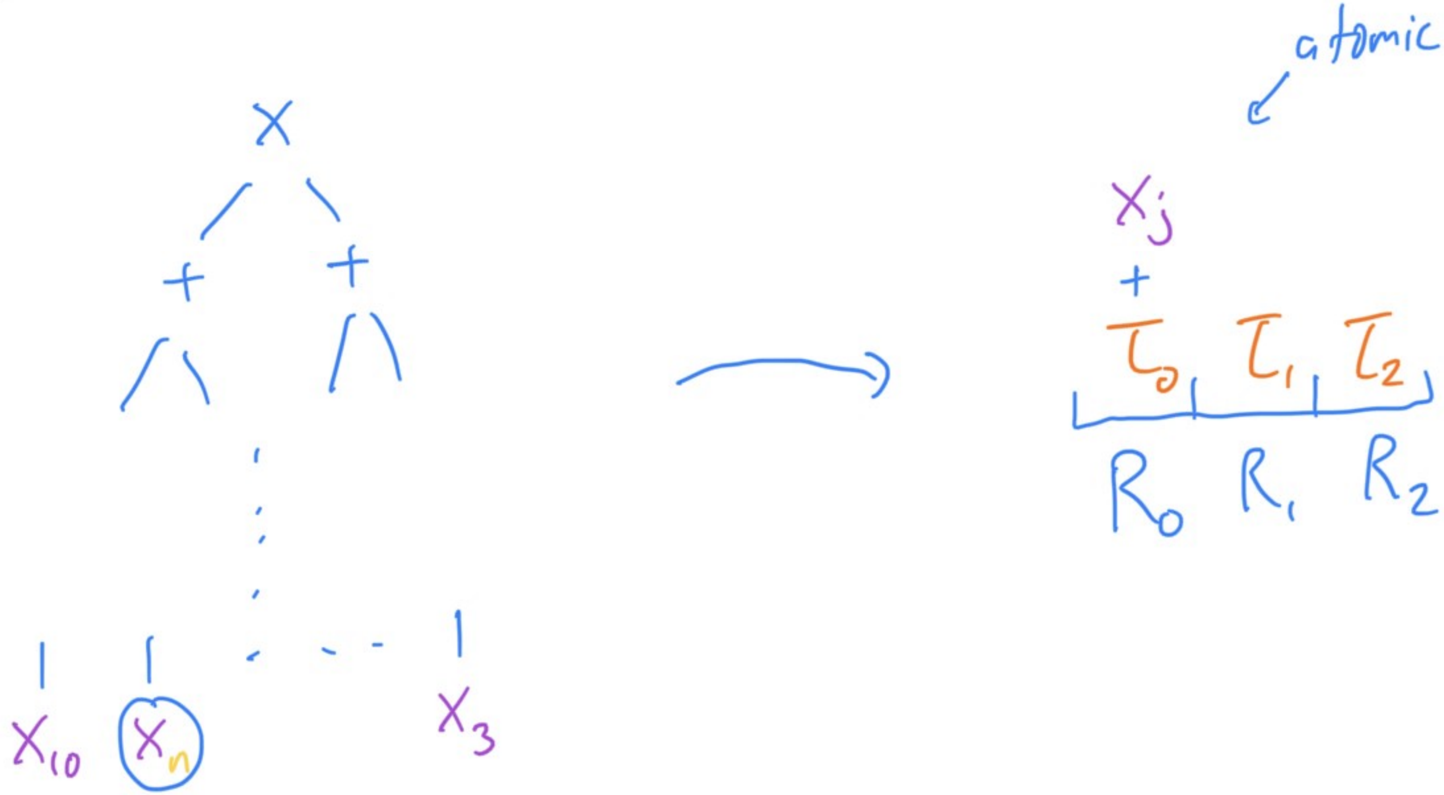


Ben-Or & Cleve



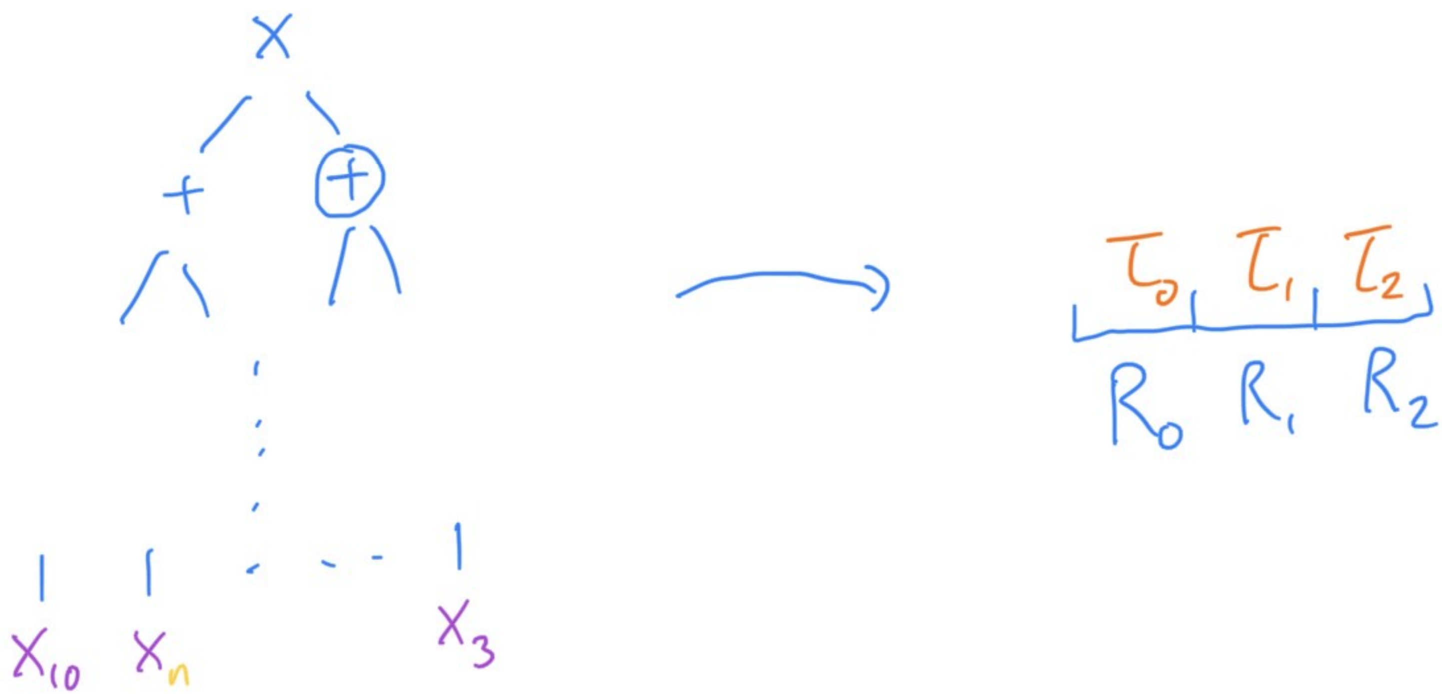
Ben-Or & Cleve

base case: $g = x_j$



BEN-OR & CLEVE

case 1 : $g = g_1 + g_2$



Ben-Or & Cleve

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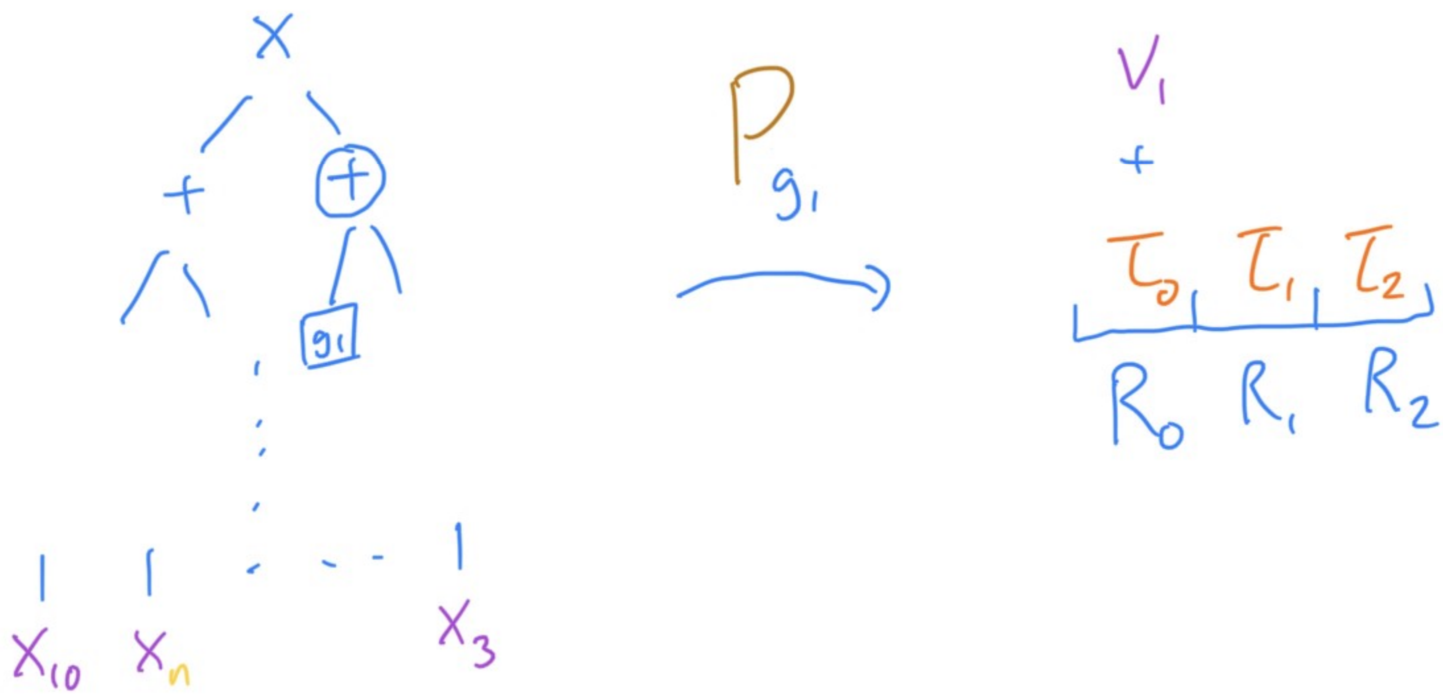
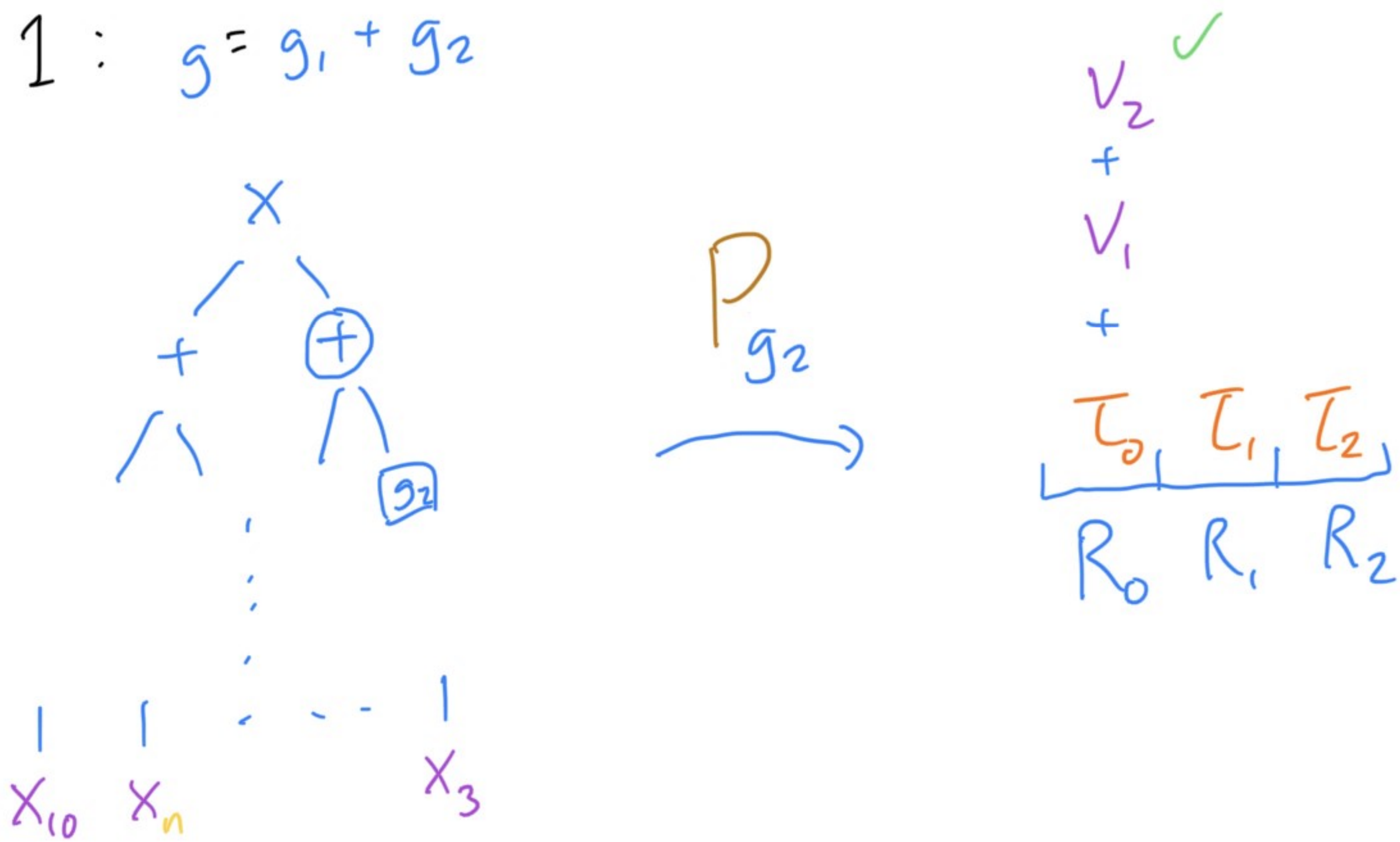


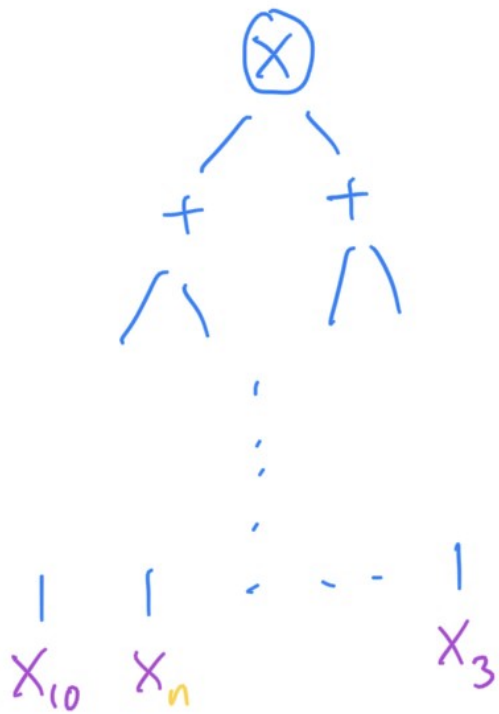
Ben-Or & Cleve

case 1 : $g = g_1 + g_2$



Ben-Or & Cleve

case 2 : $g = g_1 \times g_2$



Ben-Or & Cleve

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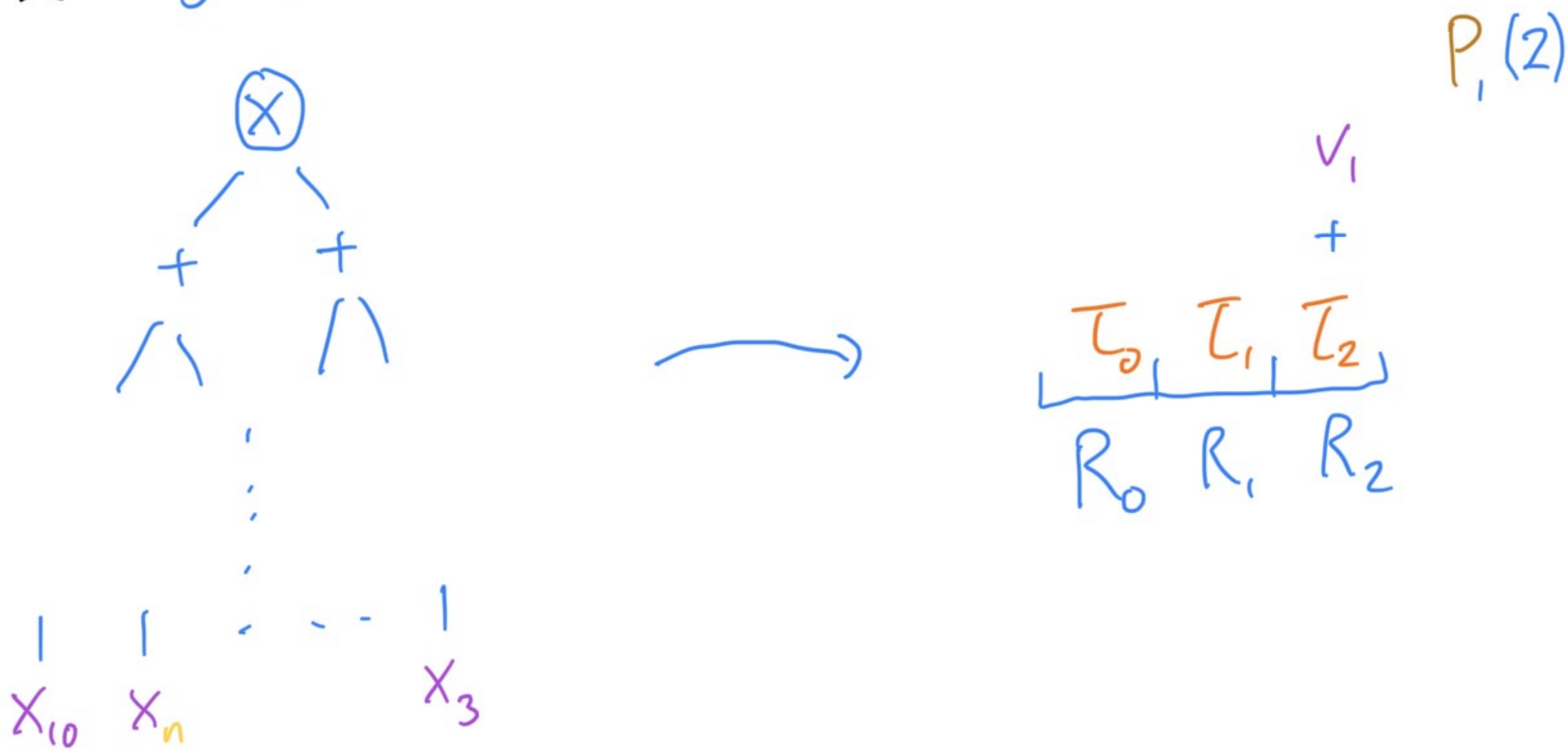

Ben-Or & Cleve

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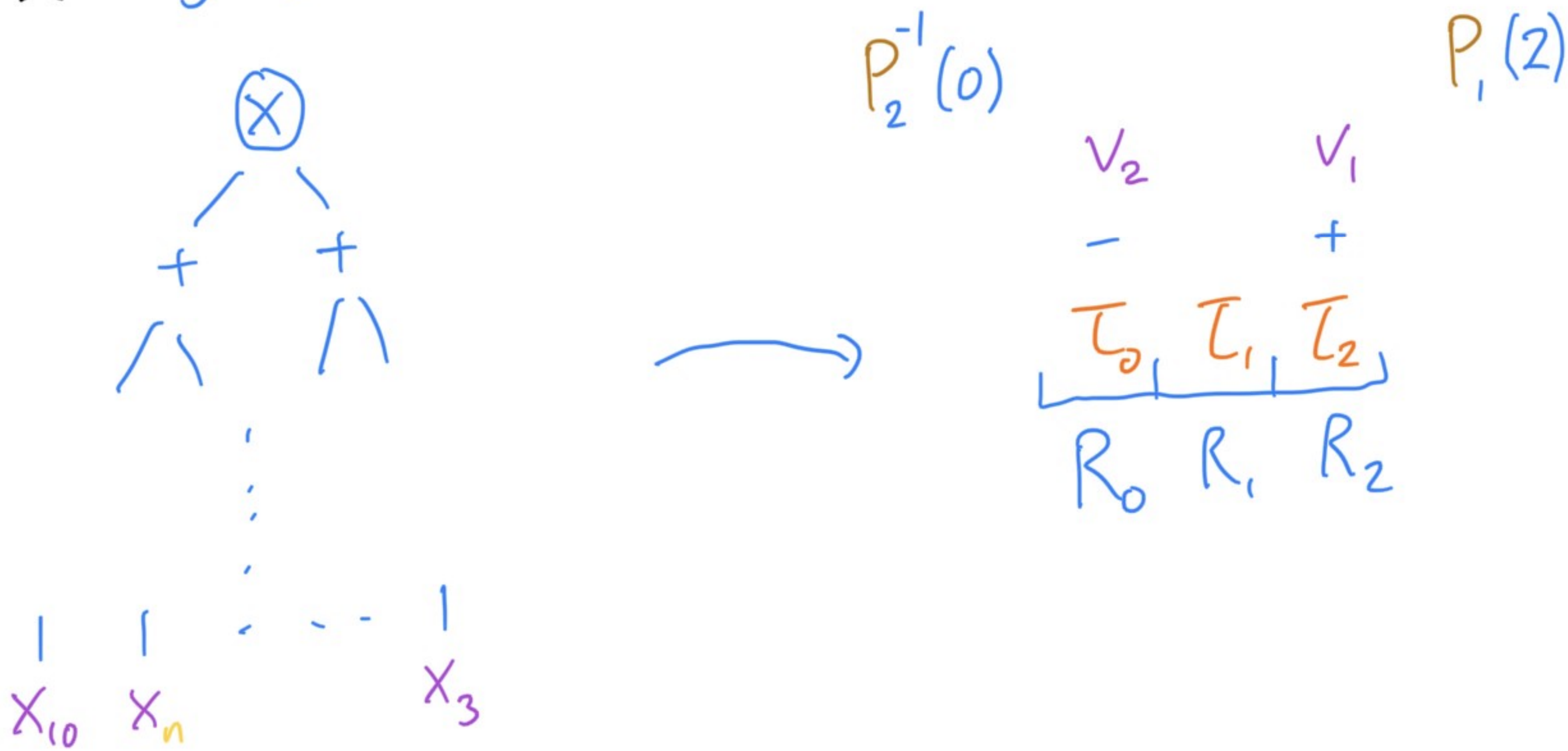


Ben-Or & Cleve

1. $P_1(1)$

2. $P_2(2)$

$R_1 = \tau_1 + v_1$

$R_2 = \tau_2 + v_2$

Ben-Or & Cleve

1. $P_1(1)$

2. $P_2(2)$

3. $R_0 \mathrel{+}= R_1 R_2$

$$R_1 = \tau_1 + v_1$$

$$R_2 = \tau_2 + v_2$$

$$R_0 = \tau_0 + (\tau_1 + v_1)(\tau_2 + v_2)$$

Ben-Or & Cleve

1. $P_1(1)$

2. $P_2(2)$

3. $R_0 \mathrel{+}= R_1 R_2$

$$R_1 = \tau_1 + v_1$$

$$R_2 = \tau_2 + v_2$$

$$R_0 = \overset{\checkmark}{\tau_0} + \overset{\times}{\tau_1 \tau_2} + \overset{\times}{v_1 \tau_2} + \overset{\times}{\tau_1 v_2} + \overset{\checkmark}{v_1 v_2}$$

Ben-Or & Cleve

4. $P_2^{-1}(2)$

5. $R_0 \mathrel{-}= R_1 R_2$

$$R_1 = \tau_1 + v_1$$

$$R_2 = \tau_2$$

$$R_0 = \overset{\checkmark}{\tau_0} + \overset{\times}{\tau_1 \tau_2} + \overset{\times}{v_1 \tau_2} + \overset{\times}{\tau_1 v_2} + \overset{\checkmark}{v_1 v_2}$$

$$- \tau_1 \tau_2 - v_1 \tau_2$$

Ben-Or & Cleve

6. $P_1^{-1}(1)$ 7. $P_2(2)$

$$R_1 = \tau_1$$

$$R_2 = \tau_2 + v_2$$

8. $R_0 \mathrel{-}= R_1 R_2$

$$\begin{aligned} R_0 = \overset{\checkmark}{\tau_0} &+ \overset{\times}{\tau_1 \tau_2} + \overset{\times}{v_1 \tau_2} + \overset{\times}{\tau_1 v_2} + \overset{\checkmark}{v_1 v_2} \\ &- \tau_1 \tau_2 - v_1 \tau_2 \\ &- \tau_1 \tau_2 \qquad\qquad - \tau_1 v_2 \end{aligned}$$

Ben-Or & Cleve

9. $P_2^{-1}(2)$

10. $R_0 \mathrel{+}= R_1 R_2$

$R_1 = \tau_1$

$R_2 = \tau_2$

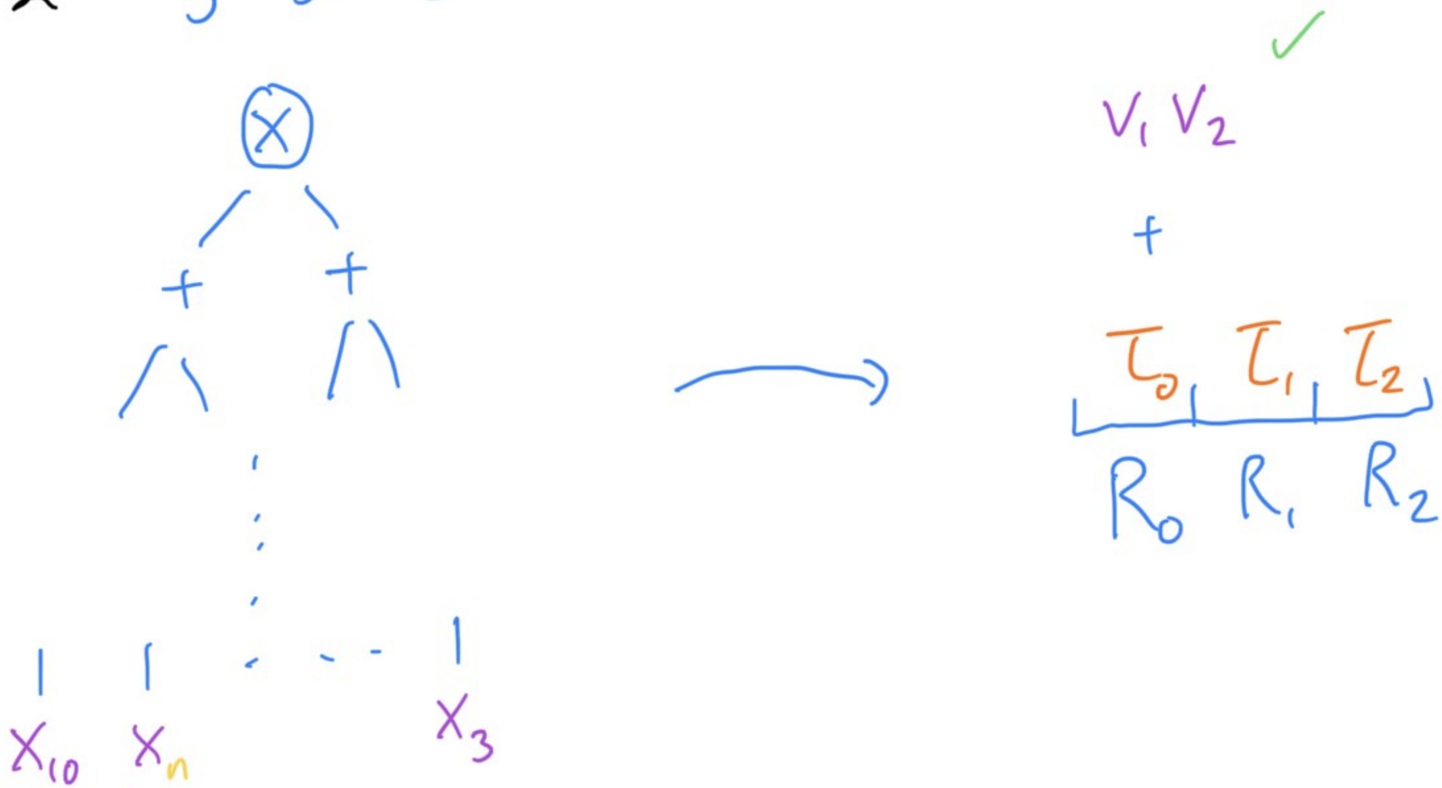
$$R_0 = \overset{\checkmark}{\tau_0} + \overset{\times}{\tau_1 \tau_2} + \overset{\times}{v_1 \tau_2} + \overset{\times}{\tau_1 v_2} + \overset{\checkmark}{v_1 v_2}$$

$$- \tau_1 \tau_2 - v_1 \tau_2$$

$$- \tau_1 \tau_2 \qquad - \tau_1 v_2 \qquad + \tau_1 \tau_2$$

Ben-Or & Cleve

case 2 : $g = g_1 \times g_2$



Ben-Or & Cleve

$$\text{space} = \underbrace{3 \log n}_{R_0 \; R_1 \; R_2} + \underbrace{O(\log n)}_{\text{misc}}$$

$+ \log$ runtime

BEN-OR & CLEVE

$$\text{space} = \underbrace{3 \log n}_{R_0\ R_1\ R_2} + \underbrace{O(\log n)}_{\text{misc}}$$

$+ \log$ runtime

$P_g \leftarrow 6$ calls to $P_{g'}$ s $+ 4$ other instructions

Ben-Or & Cleve

$$\text{space} = \underset{R_0\ R_1\ R_2}{3 \log n} + \underset{\text{misc}}{O(\log n)}$$

$$+ \log \text{runtime}$$

$$P_g \leftarrow 6 \text{ calls to } P_{g'}\text{s} + 4 \text{ other instructions}$$

$$R(h) \leq 6 \cdot R(h-1) + 4 \rightarrow 6^{\log n} \ \text{poly}\ n \ \text{instructions} \checkmark \qquad \square$$

$$\mathrm{SPACE}(f, z) = \mathrm{SPACE}(f) + |z| \; ?$$

$$\mathrm{SPACE}(f, z) = \mathrm{SPACE}(f) + |z| \; ?$$

[CMWBS'12]: YES for most f (conjecture)

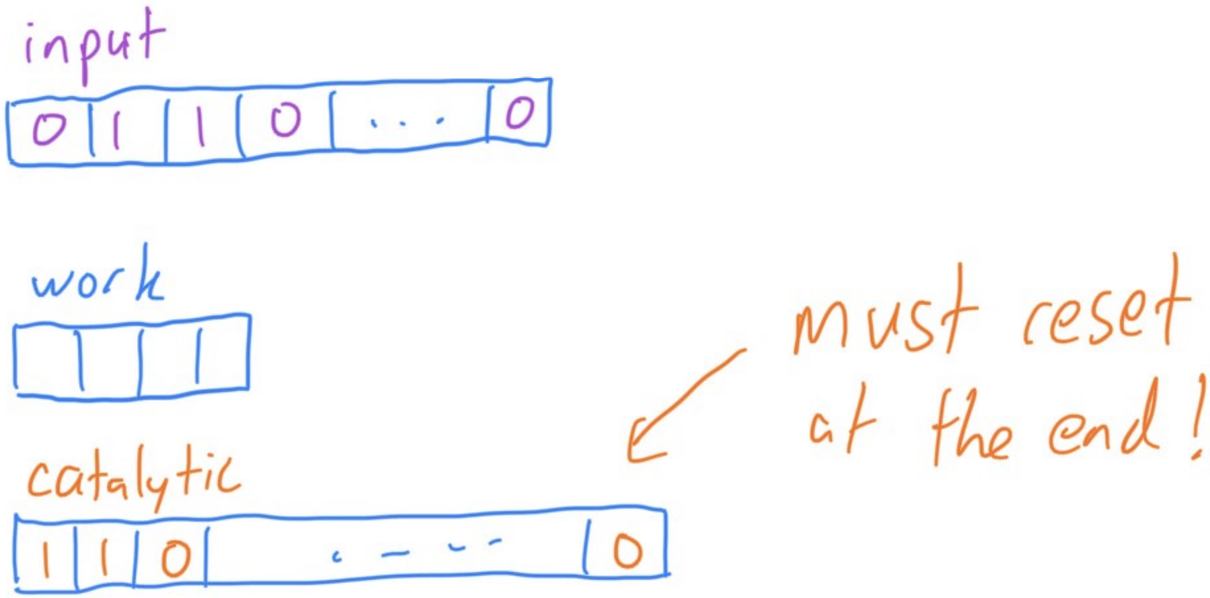
[BC'89]: NO for $f = +, \times$

CATALYTIC COMPUTING

[BCKLS'14]: let's model it

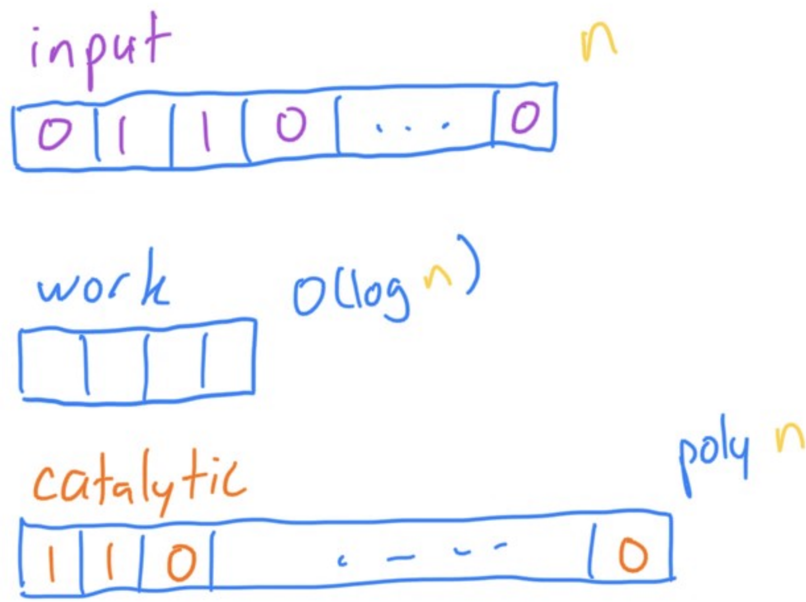
CATALYTIC COMPUTING

[BCKLS'14]: let's model it



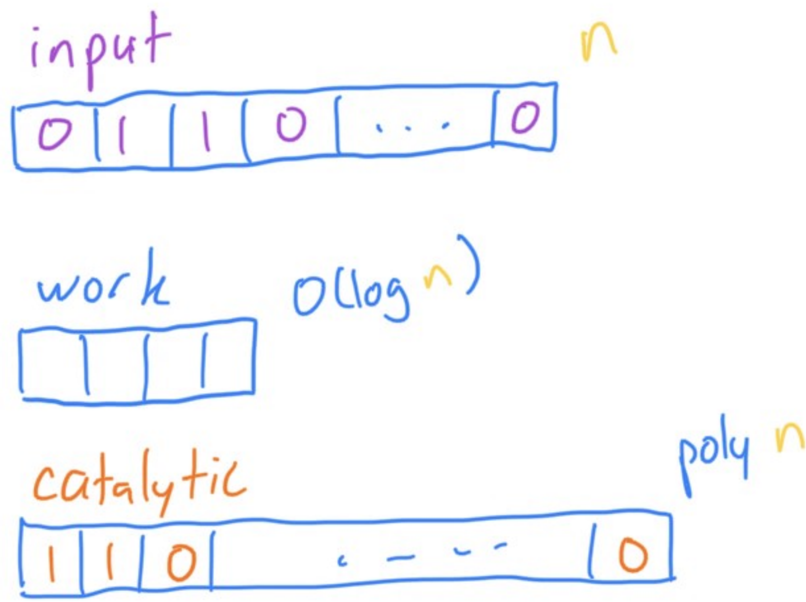
CATALYTIC COMPUTING

CL



CATALYTIC COMPUTING

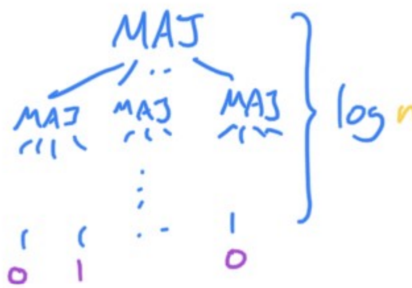
CL



Q: what can CL do that L can't?

CATALYTIC COMPUTING

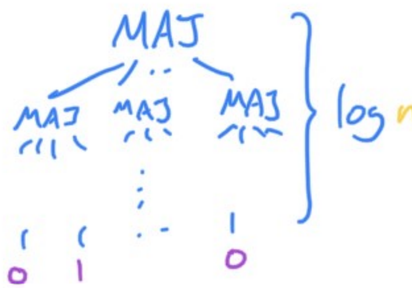
TC^1



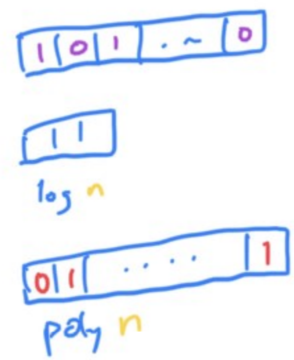
$$\begin{array}{ccccccccc} & & & \subseteq & TC^1 & \subseteq & & & \\ NC^1 & \subseteq & L & \subseteq & NL & \subseteq & NC^2 & \subseteq & P \end{array}$$

CATALYTIC COMPUTING

TC^1



CL



$$TC^1 \subseteq CL$$

$$NL \subseteq TC^1 \subseteq NC^2$$

$$NC^1 \subseteq L \subseteq NL \subseteq NC^2 \subseteq P$$

CATALYTIC COMPUTING

LEMMA: $\forall g \in \mathcal{C}$, $\exists P_g$ s.t.

$$P_g : \quad R_0 \leftarrow R_0 + V_g$$
$$R_i \leftarrow R_i \qquad \forall i \neq 0$$

gateset: $MAJ(x) = \begin{cases} 1 & \text{if } \sum_{i=1}^{m} x_i \geq \frac{m}{2} \\ 0 & \text{o.w.} \end{cases}$

CATALYTIC COMPUTING

$$MAJ(x) \equiv \sum_{k=\frac{m}{2}}^{m} \left[1 - \left(\sum_{i=1}^{m} x_i - k \right)^{p-1} \right] \bmod p$$

CATALYTIC COMPUTING

$$\mathrm{MAJ}(x) \equiv \sum_{k=\frac{m}{2}}^{m} \left[1 - \left(\sum_{i=1}^{m} x_i - k \right)^{p-1} \right] \bmod p$$

$$P_{\Sigma} \qquad\qquad P_{\wedge p-1}$$

CATALYTIC COMPUTING

$$\mathrm{MAJ}(x) \equiv \sum_{k=\frac{m}{2}}^{m} \left[1 - \left(\sum_{i=1}^{m} x_i - k \right)^{p-1} \right] \bmod p$$

$$P_{\Sigma} \qquad\qquad P_{\wedge p-1}$$

Efficiency: poly n registers

$O(1)$ recursive calls

$\square$

$$\mathrm{SPACE}(f, z) = \mathrm{SPACE}(f) + |z| \; ?$$

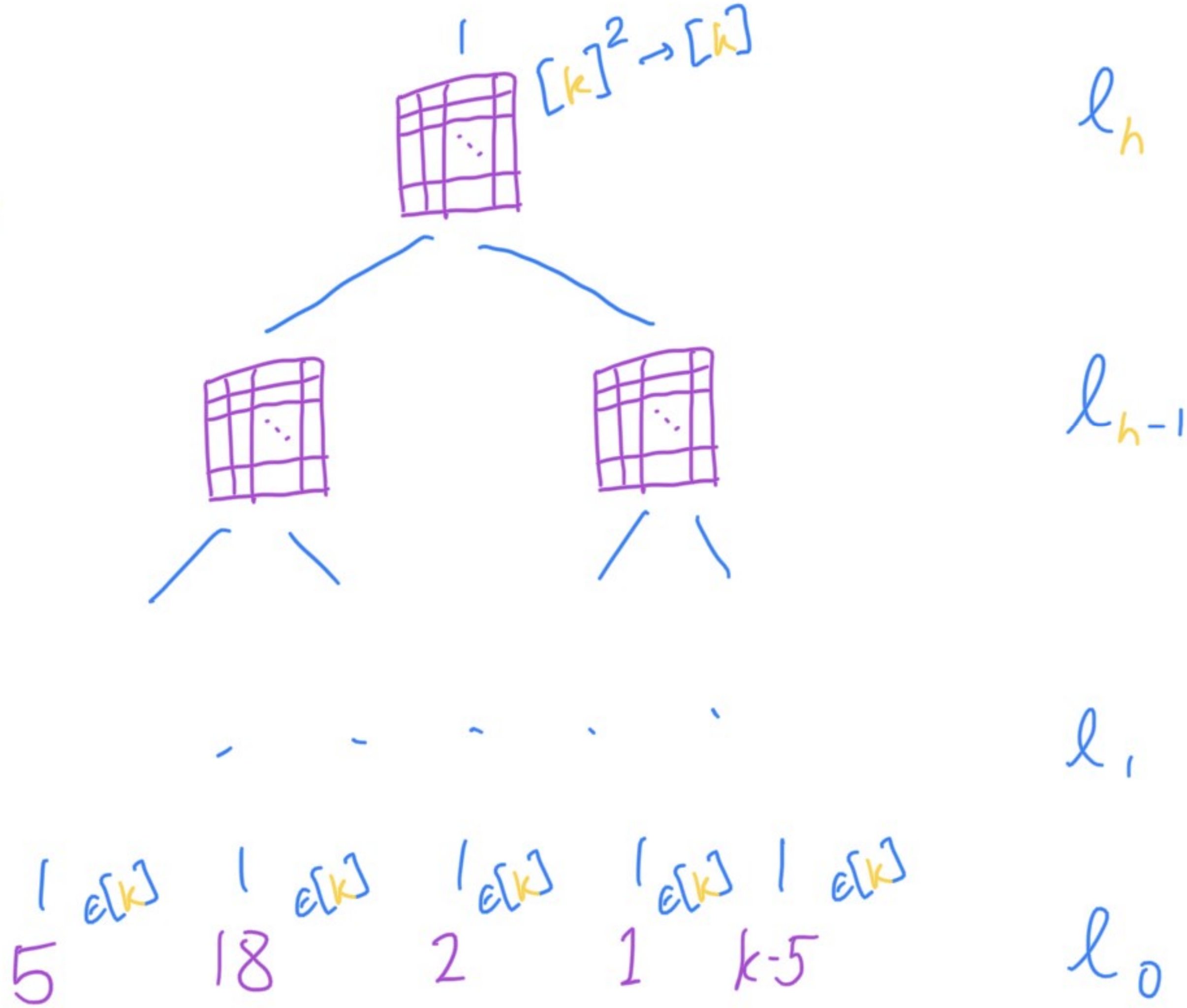
TEP UPPER BOUNDS

<u>THEOREM</u> [CM'20, 21]: $TEP_{k,h}$ can be solved in space $O(h \log k / \log h)$

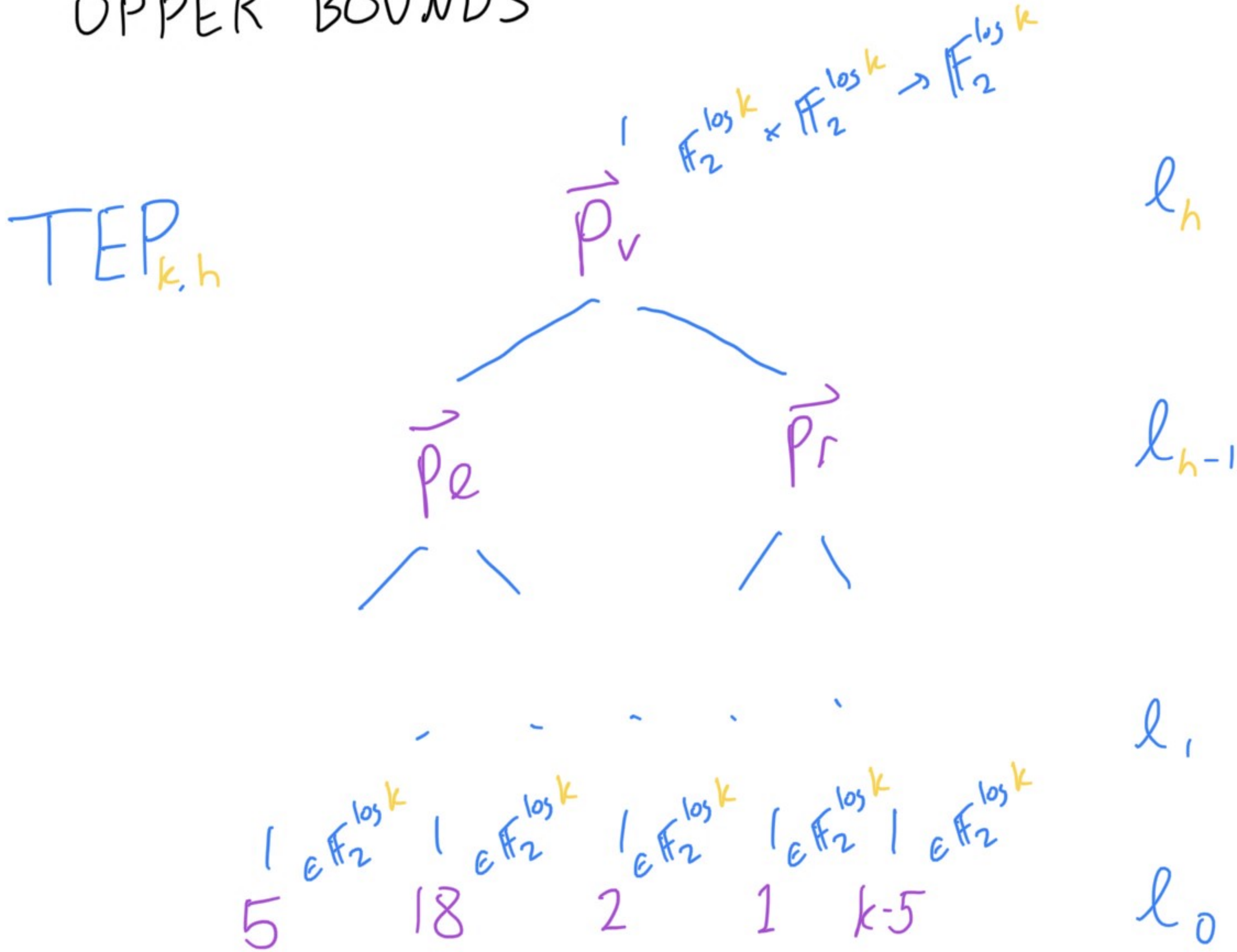
$(= O(\log^2 n / \log \log n))$

TEP UPPER BOUNDS

$TEP_{k,h}$



TEP UPPER BOUNDS



TEP UPPER BOUNDS

<u>LEMMA</u>: $\forall g \in \mathcal{C}$, $\exists P_g$ s.t.

$$P_g : \quad R_0 \leftarrow R_0 + v_g$$

$$R_i \leftarrow R_i \qquad \forall i \neq 0$$

gateset: $\vec{p}(\vec{y}, \vec{z}) : \mathbb{F}_2^{\log k} \times \mathbb{F}_2^{\log k} \to \mathbb{F}_2^{\log k}$

TEP UPPER BOUNDS

$$\vec{p}(\vec{y}, \vec{z}) : \mathbb{F}_2^{\log k} \times \mathbb{F}_2^{\log k} \to \mathbb{F}_2^{\log k}$$

Efficiency: $3 \log k$ registers over $\mathbb{F}_2$

$\to$ space $= 3 \log k$

TEP UPPER BOUNDS

$$\vec{p}(\vec{y}, \vec{z}) : \mathbb{F}_2^{\log k} \times \mathbb{F}_2^{\log k} \to \mathbb{F}_2^{\log k}$$

Efficiency: $3 \log k$ registers over $\mathbb{F}_2$

k^2 recusive calls

$\to$ space $= 3 \log k + \log (k^2)^h = h \log k$ $\cancel{\square}$

TEP UPPER BOUNDS

$$\vec{p}(\vec{y}, \vec{z}) : \mathbb{F}_2^{\log k} \times \mathbb{F}_2^{\log k} \to \mathbb{F}_2^{\log k}$$

Efficiency: $3 \log k$ registers over $\mathbb{F}_2$

$2^{\deg(p)}$ recusive calls

TEP UPPER BOUNDS

FEWER RECURSIVE CALLS $2^{o(\deg(p))}$

↓

FASTER TEP ALGORITHMS $\left(2^{o(\log k)}\right)^h$

↓

SMALLER SPACE TEP $o(h \log k)$

TEP UPPER BOUNDS

<u>THEOREM</u> [CM'23]: $\mathrm{TEP}_{k,h}$ can be solved in space $O(h + \log k) \cdot \log\log k$

$(= O(\log n \cdot \log\log n))$

TEP UPPER BOUNDS

$\vec{p}_v(\vec{\ell}, \vec{r})$: $O(\deg(p))$ recusive calls

$$\text{space} = \underbrace{\log\left[O(\log k)^h\right]}_{\text{runtime}}$$

TEP UPPER BOUNDS

$\vec{p}_v(\vec{l}, \vec{r})$: $O(\deg(p))$ recusive calls

$3 \log k$ registers over $\mathbb{F}_{O(\deg(p))}$

$$\text{space} = \underbrace{\log\left[O(\log k)^h\right]}_{\text{runtime}} + \underbrace{(3 \log k)}_{\#\text{ reg.}} \cdot \underbrace{\log\left[O(\log k)\right]}_{\text{size per reg.}}$$

$$= O(h + \log k) \cdot \log\log k$$

TEP UPPER BOUNDS

PROOF

K

TEP upper bounds

Proof

K

ω_m m^{-1}

TEP UPPER BOUNDS

PROOF

$$\omega_m$$

1) $\omega_m^m = 1$

2) $\sum_{j=1}^{m} \omega_m^j = 0$

TEP UPPER BOUNDS

PROOF

$$\omega_m$$

1) $\omega_m^m = 1$

2) $\sum_{j=1}^{m} \omega_m^{jb} = 0 \quad m \nmid b$

TEP UPPER BOUNDS

PROOF

$$\omega_m$$

1) $\omega_m^m = 1$

2) $\sum_{j=1}^{m} \omega_m^{jb} = \begin{cases} m & b=0 \\ 0 & \text{o.w.} \end{cases} \qquad 0 \leq b < m$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} \prod_{i=1}^{d} \left(\omega_m^j \tau_i + v_i \right)$$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} \prod_{i=1}^{d} \left(\omega_m^j \tau_i + v_i \right)$$

$$\sum_{j=1}^{m} \sum_{b=0}^{d} \sum_{|S|=b} \left(\prod_{i \in S} \omega_m^j \tau_i \right) \left(\prod_{i \notin S} v_i \right)$$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} \sum_{b=0}^{d} \sum_{|S|=b} \left(\prod_{i \in S} \omega_m^j \tau_i \right) \left(\prod_{i \notin S} v_i \right)$$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} \sum_{b=0}^{d} \sum_{|S|=b} \left(\prod_{i \in S} \omega_m^j \tau_i \right) \left(\prod_{i \notin S} v_i \right)$$

$$\sum_{b=0}^{d} \left(\sum_{j=1}^{m} \omega_m^{jb} \right) \left[\sum_{|S|=b} \left(\prod_{i \in S} \tau_i \right) \left(\prod_{i \notin S} v_i \right) \right]$$

TEP UPPER BOUNDS

PROOF

$$\sum_{b=0}^{d}\left(\sum_{j=1}^{m} \omega_m^{jb}\right)\left[\sum_{|S|=b}\left(\prod_{i\in S} \tau_i\right)\left(\prod_{i\notin S} v_i\right)\right]$$

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PROOF

$$\sum_{b=0}^{d}\left(\sum_{j=1}^{m}\omega_m^{jb}\right)\left[\sum_{|S|=b}\left(\prod_{i\in S}\tau_i\right)\left(\prod_{i\notin S}v_i\right)\right]$$

$$\sum_{j=1}^{m}\omega_m^{jb} = \begin{cases} m & b=0 \\ 0 & \text{o.w.} \end{cases} \qquad 0 \le b < m$$

TEP UPPER BOUNDS

PROOF

$$\sum_{b=0}^{d}\left(\sum_{j=1}^{m} \omega_m^{jb}\right)\left[\sum_{|S|=b}\left(\prod_{i\in S}\tau_i\right)\left(\prod_{i\notin S} v_i\right)\right]$$

$$\sum_{j=1}^{m}\prod_{i=1}^{d}\left(\omega_m^{j}\tau_i + v_i\right) = m\cdot\prod_{i=1}^{d} v_i$$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} m^{-1} \prod_{i=1}^{d} \left(\omega_m^j \tau_i + v_i \right) = \prod_{i=1}^{d} v_i$$

TEP UPPER BOUNDS

PROOF

$$\sum_{j=1}^{m} m^{-1} \prod_{i=1}^{d} (\omega_m^j \tau_i + v_i) = \prod_{i=1}^{d} v_i$$

$$\sum_{j=1}^{m} m^{-1} p(\overrightarrow{\omega_m^j \tau_i + v_i}) = p(\vec{v_i})$$

same logic for each monomial in p

TEP UPPER BOUNDS

Program for $p(x_1 \ldots x_n)$

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Choose $K = \mathbb{F}_{2^r}$ for $r > \log \deg(p) + 1$

$m = 2^r - 1$ $\qquad$ $m^{-1} = m \equiv -1$

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ω_m = any generator of mult. group of K

registers $R_i \in K$

TEP UPPER BOUNDS

Program for $p(x_1 \ldots x_n)$

for $j = 1 \ldots m$:

$R_i \leftarrow \omega_m^j R_i \quad \forall i$

$P_x \quad (R_i \mathrel{+}= x_i \quad \forall i)$

$$R_i = \omega_m^j \tau_i + v_i$$

TEP UPPER BOUNDS

Program for $p(x_1 \ldots x_n)$

for $j = 1 \ldots m$:

$$P_x^{-1} \quad (R_i \mathrel{-}= x_i \quad \forall i)$$

$$R_i \leftarrow \omega_m^{-j} R_i \quad \forall i$$

$$R_i = \omega_m^j \tau_i$$

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Program for $p(x_1 \ldots x_n)$

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$P_x \quad (R_i \mathrel{+}= x_i \quad \forall i)$ $\qquad$ $R_i \leftarrow \omega_m^{-j} R_i \quad \forall i$

$$R^{out} \leftarrow R^{out} + m^{-1} p(R_1 \ldots R_n)$$

TEP UPPER BOUNDS

$$\vec{p}_v : \{0,1\}^{\log k} \times \{0,1\}^{\log k} \to \{0,1\}^{\log k}$$

TEP UPPER BOUNDS

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$$|K| \approx 2 \log k$$

SPACE: $3 \log k \cdot \log |K|$

TIME: $(2|K|)^h$

TEP UPPER BOUNDS

$$\vec{p}_v : \{0,1\}^{\log k} \times \{0,1\}^{\log k} \to \{0,1\}^{\log k}$$

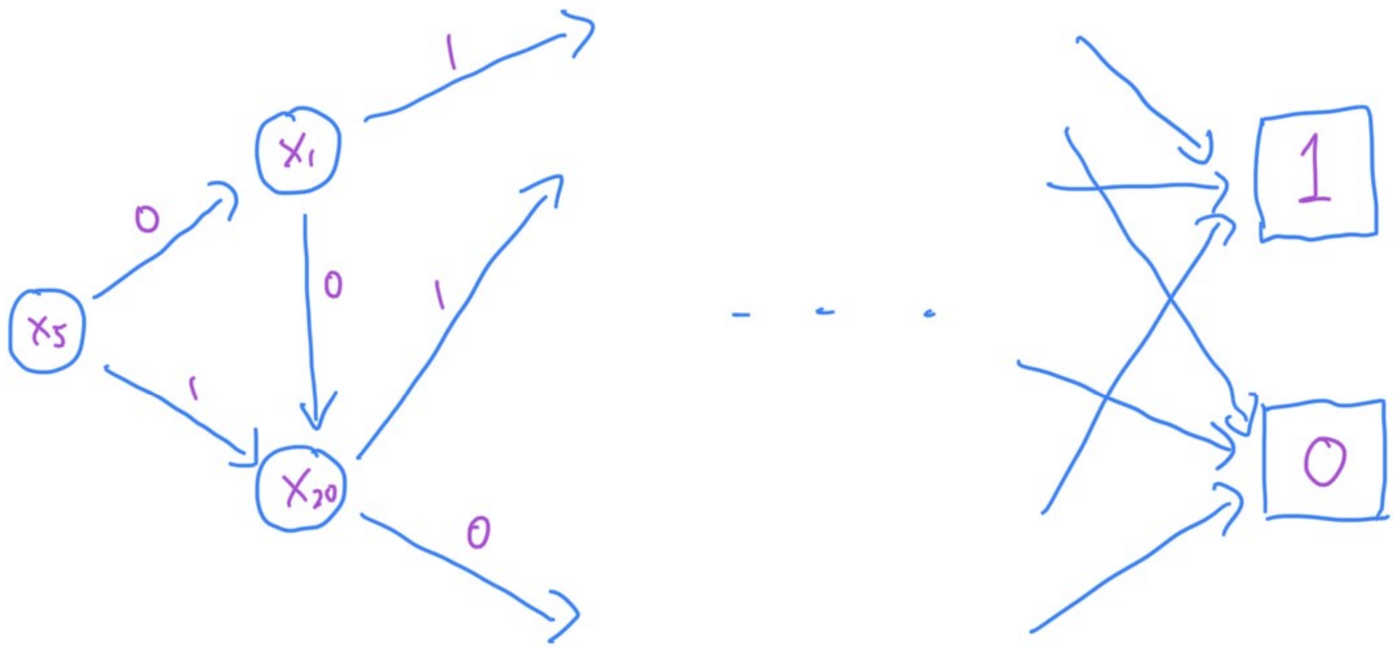
$$|K| \approx 2 \log k$$

SPACE: $3 \log k \cdot \log |K|$

TIME: $(2|K|)^h$

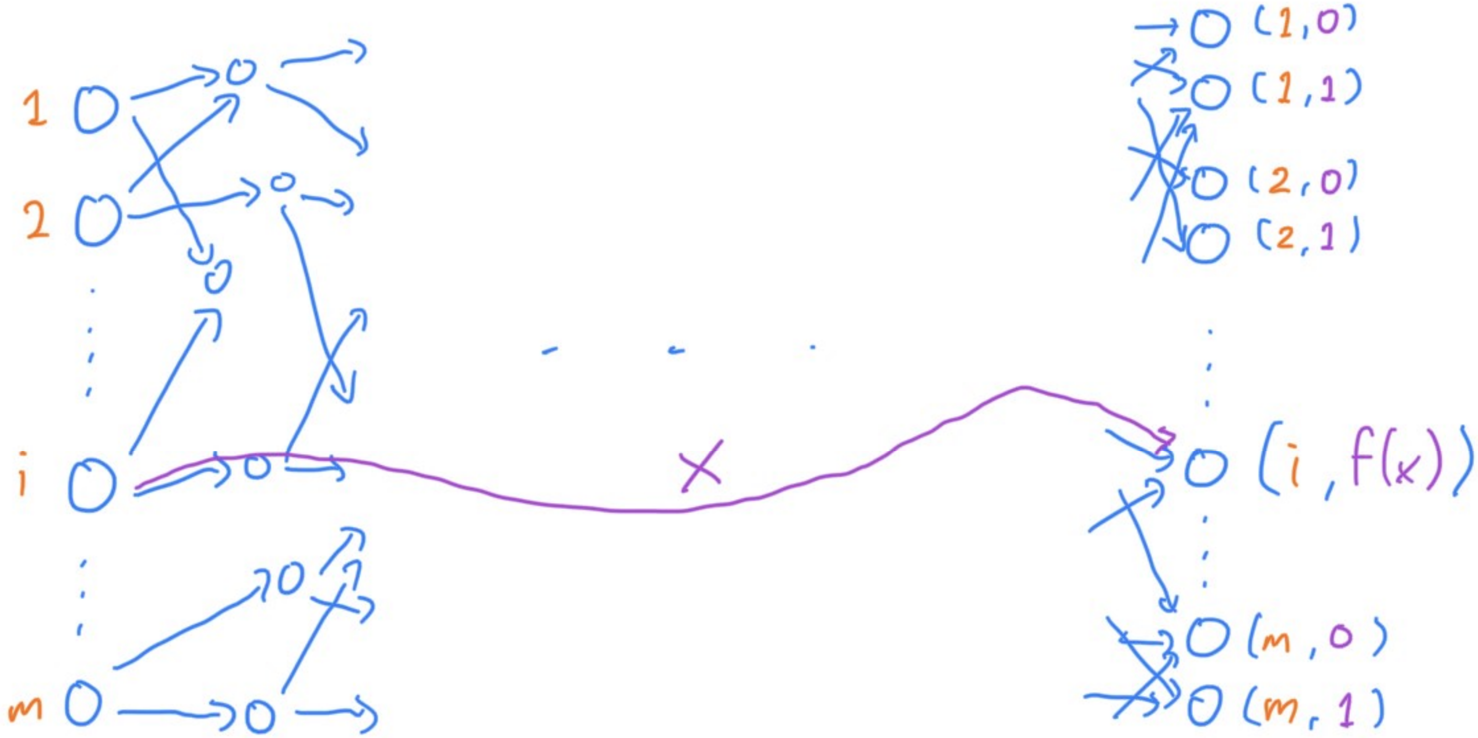
TOTAL: $O(h + \log k) \cdot \log \log k$ $\square$

AMORTIZED BPs



size s $\equiv$ non-uniform space $\log s$

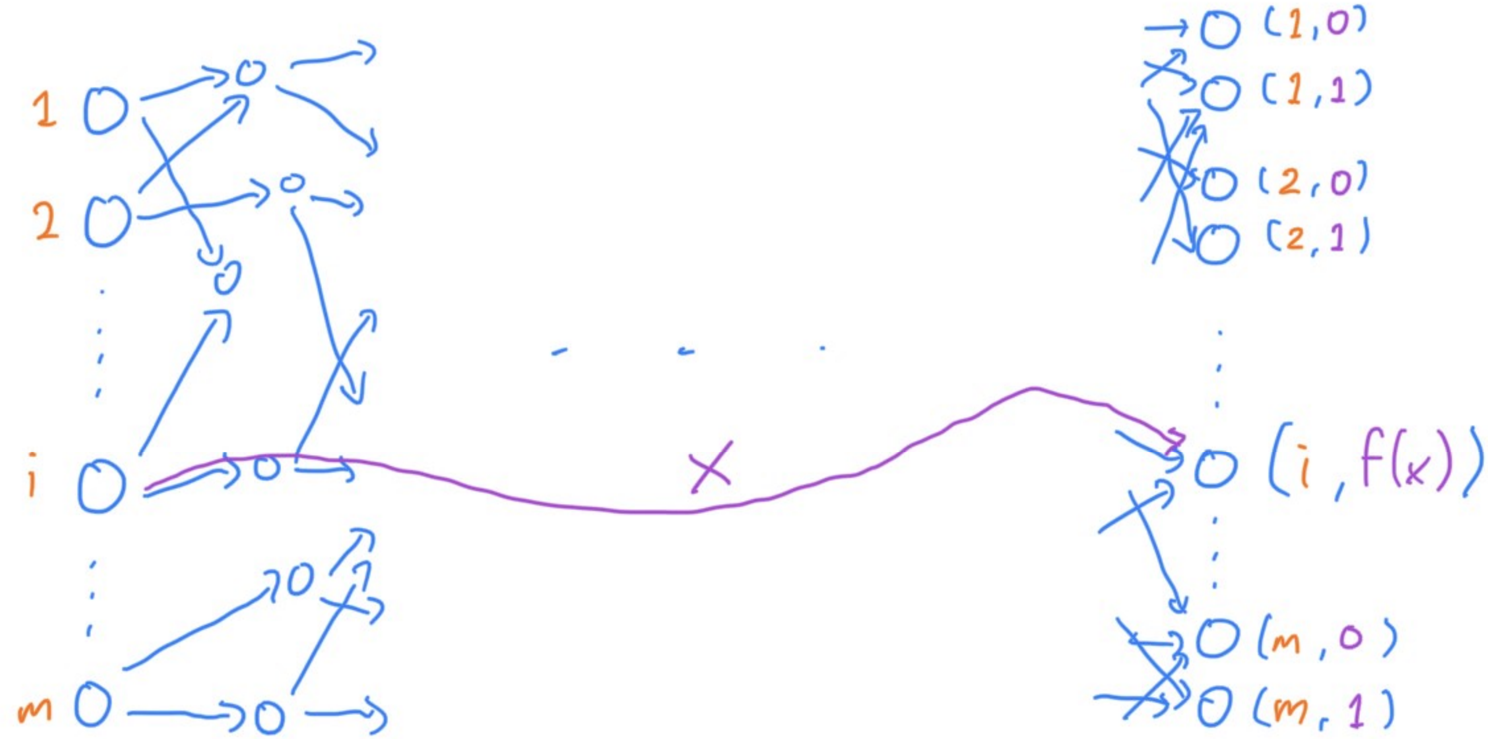
AMORTIZED BPs



catalytic $\log m$

size mS $\equiv$ non-uniform space $\log S$

AMORTIZED BPs



size ms $\equiv$ amortized$_m$ size s

AMORTIZED BPs

THEOREM [P'17]: $s = O(n) \qquad m = 2^{2^n}$

Amortized BPs

Theorem [P'17] : $s = O(n)$ $\quad m = 2^{2^n}$

Theorem [CM'22] : $s = O_\varepsilon(n)$ $\quad m = 2^{2^{\varepsilon n}}$

AMORTIZED BPs

THEOREM [P'17]: $s = O(n)$ $m = 2^{2^n}$

THEOREM [CM'22]: $s = O_\varepsilon(n)$ $m = 2^{2^{\varepsilon n}}$

THEOREM [CM'23]: $s = n^{2+\varepsilon}$ $m = 2^{O_\varepsilon(n)}$

KRW AND SPACE

<u>CONJECTURE</u> [KRW'95]: $TEP_{d,h} \notin NC^1$

KRW and Space

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Theorem [CM'23]: $KRW \to NC^1 \neq L$

KRW AND SPACE

1. KRW gives a very sharp separation between complexity classes

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2. KRW $\to$ quasipoly (uniform) separation between formulas and branching programs

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1. KRW gives a very sharp separation between complexity classes

2. KRW $\rightarrow$ quasipoly (uniform) separation between formulas and branching programs

3. formally easier to show $STCONN \notin NC^1$ than $TEP \notin NC^1$

WHAT NOW?

- TEP still not in L yet!

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- what is TEP complete for?

WHAT NOW?

- TEP still not in L yet!
- what is TEP complete for?
- other things to do with catalytic?

SHAMELESS PLUG

SURVEY [M'23]: B. EATCS
ECCC

T H A N K S !