Mathematical Analysis 1:
Tutorial #12

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Exercise 1. *Determine which of the following integrals are improper, and explain why. (You do not need to evaluate the integrals.)*

(a) $\int_{-1}^{1} \ln|x|dx$;

(b) $\int_{-1}^{1} \ln(1+x^2)dx$;

(c) $\int_{0}^{-1} \frac{dx}{1+x^2}$;

(d) $\int_{-\infty}^{2} e^x dx$;

(e) $\int_{-\infty}^{\infty} x dx$;

(f) $\int_{-5}^{18} \frac{dx}{x^2+x-6}$;

(g) $\int_{10}^{100} \frac{dx}{x^2+x-6}$;

(h) $\int_{-\pi/2}^{\pi/2} \tan x dx$;

(i) $\int_{-\pi/2}^{\pi/2} \arctan x dx$.

Exercise 2. *Compute the values of the improper integrals below (if they exist), and state whether the improper integrals are convergent or divergent.*

(a) $\int_{0}^{1} \frac{dx}{x\sqrt{x}}$;

(b) $\int_{0}^{+\infty} e^{-x/2} dx$;

(c) $\int_{-\infty}^{+\infty} xe^{-x^2} dx$;

(d) $\int_{0}^{1} \frac{dx}{2x-1}$;

(e) $\int_{-\infty}^{\infty} \cos(\pi x) dx$;

(f) $\int_{0}^{2} \frac{dx}{x \ln x}$;

(g) $\int_{0}^{7} \frac{dx}{\sqrt{7-x}}$;

(h) $\int_{0}^{+\infty} \frac{dx}{\sqrt{x}(1+x)}$.

Exercise 3. *Find the values of the real constant p for which the following improper integrals converge:*

(a) $\int_{0}^{1} \frac{dx}{x^p}$;

(b) $\int_{0}^{1} x^p \ln x dx$;

(c) $\int_{e}^{+\infty} \frac{dx}{x(\ln x)^p}$.

Exercise 4. *Compute the area between the curves $y = \sin x$ and $y = e^x$, and between $x = 0$ and $x = \frac{\pi}{2}$.*

Exercise 5. *Compute the area between the curves $y = \tan x$ and $y = 2\sin x$, and between $x = -\frac{\pi}{3}$ and $x = \frac{\pi}{3}$.*