Mathematical Analysis 1:
Tutorial #11 - some solutions

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Exercise 3. *Compute the following definite integrals:*

(a) $\int\limits_{-1}^{0} \frac{dx}{1-\sqrt[3]{x}}$; *(b)* $\int\limits_{0}^{\pi/2} x\cos x dx$; *(c)* $\int\limits_{0}^{\pi/2} e^{2x}\cos x dx$.

Solution. (a)

$u = 1-\sqrt[3]{x}, \quad du = -\frac{1}{3}x^{-2/3}dx$

$dx = -3x^{2/3}du = -3(1-u)^2 du$

$x=-1, \quad x=0$

$u=2, \quad u=1$

$$\begin{aligned}
\int\limits_{-1}^{0} \frac{dx}{1-\sqrt[3]{x}} &= \int\limits_{2}^{1} \frac{-3(1-u)^2}{u}du \\
&= -3\int\limits_{2}^{1} \frac{(1-u)^2}{u}du \\
&= 3\int\limits_{1}^{2} \frac{(1-u)^2}{u}du \\
&= 3\int\limits_{1}^{2} \frac{1-2u+u^2}{u}du \\
&= 3\int\limits_{1}^{2} \frac{1}{u}-2+u\ du \\
&= 3\Big(\ln|u|-2u+\tfrac{1}{2}u^2\Big)\Big|_1^2 \\
&= 3\Big((\ln 2-4+2)-(\ln 1-2+\tfrac{1}{2})\Big) \\
&= 3\big(\ln 2-\tfrac{1}{2}\big)
\end{aligned}$$

(b)

$$\int_0^{\pi/2} x\cos x dx = (x\sin x)\Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x dx \qquad \begin{array}{ll} u = x, & v = \sin x \\ du = dx, & dv = \cos x dx \end{array}$$

$$= (x\sin x)\Big|_0^{\pi/2} - (-\cos x)\Big|_0^{\pi/2}$$

$$= (x\sin x + \cos x)\Big|_0^{\pi/2}$$

$$= \left(\tfrac{\pi}{2}\sin(\tfrac{\pi}{2}) + \cos(\tfrac{\pi}{2})\right) - \left(0\sin 0 + \cos 0\right)$$

$$= \tfrac{\pi}{2} - 1$$

(c) In what follows, color coding is for emphasis (the integral that we need to evaluate is in red).

$$\int_0^{\pi/2} e^{2x}\cos x dx \qquad \begin{array}{ll} u_1 = e^{2x}, & v_1 = \sin x \\ du_1 = 2e^{2x}dx, & dv_1 = \cos x dx \end{array}$$

$$= \left(e^{2x}\sin x\right)\Big|_0^{\pi/2} - \int_0^{\pi/2} 2e^{2x}\sin x dx$$

$$= e^{\pi} - 2\int_0^{\pi/2} e^{2x}\sin x dx \qquad \begin{array}{ll} u_2 = e^{2x}, & v_2 = -\cos x \\ du_2 = 2e^{2x}dx, & dv_2 = \sin x dx \end{array}$$

$$= e^{\pi} - 2\left((-e^{2x}\cos x)\Big|_0^{\pi/2} - \int_0^{\pi/2} -2e^{2x}\cos x dx\right)$$

$$= e^{\pi} + 2\left((e^{2x}\cos x)\Big|_0^{\pi/2} - 2\int_0^{\pi/2} e^{2x}\cos x dx\right)$$

$$= e^{\pi} + 2\left(-1 - 2\int_0^{\pi/2} e^{2x}\cos x dx\right)$$

$$= e^{\pi} - 2 - 4\int_0^{\pi/2} e^{2x}\cos x dx$$

We have now obtained

$$\int_0^{\pi/2} e^{2x}\cos x dx = e^{\pi} - 2 - 4\int_0^{\pi/2} e^{2x}\cos x dx.$$

By solving for $\int_0^{\pi/2} e^{2x}\cos x dx$, we get

$$\int_0^{\pi/2} e^{2x}\cos x dx = \tfrac{e^{\pi}-2}{5}.$$

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