Mathematical Analysis 1:
Tutorial #11

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Exercise 1. *Using the Fundamental Theorem of Calculus, Part I, compute the* ***derivatives*** *of the following:*

(a) $\int_{\sqrt{217}}^{e^{2x}} \frac{dt}{t^2}$; (b) $\int_{2x}^{0} \sin t dt$; (c) $\int_{x^2}^{x^4} \ln t dt$.

Exercise 2. *Compute the following definite integrals by first computing the corresponding indefinite integrals, and then applying the Fundamental Theorem of Calculus, Part II:*

(a) $\int_{4}^{9} \frac{\ln x}{\sqrt{x}} dx$; (b) $\int_{0}^{1} xe^x dx$; (c) $\int_{0}^{1} \frac{e^x+1}{e^x+x} dx$.

Exercise 3. *Compute the following definite integrals:*

(a) $\int_{-1}^{0} \frac{dx}{1-\sqrt[3]{x}}$; (b) $\int_{0}^{\pi/2} x \cos x dx$; (c) $\int_{0}^{\pi/2} e^{2x} \cos x dx$.

Exercise 4. *Let $a, b \in \mathbb{R}$ be such that $a < b$, and let $f : [a, b] \to \mathbb{R}$ be a continuous function. Prove that there exists some $c \in [a, b]$ such that $\int_{a}^{b} f(x)dx = f(c)(b - a)$.*

Exercise 5.

(a) *Prove that*

$$\lim_{x\to 0} \Big(\int_{0}^{x^2} \sin(\sqrt{t})dt \Big) = 0.$$

(b) *Compute*

$$\lim_{x\to 0^+} \Big(\frac{1}{x^3} \int_{0}^{x^2} \sin(\sqrt{t})dt \Big) \qquad \text{and} \qquad \lim_{x\to 0^-} \Big(\frac{1}{x^3} \int_{0}^{x^2} \sin(\sqrt{t})dt \Big).$$

Exercise 6. *Let $a > 0$ be a real number, and let $f : [-a, a] \to \mathbb{R}$ be a continuous function. Prove the following:*

(a) *if f is even, then $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$;*

(b) *if f is odd, then $\int_{-a}^{a} f(x)dx = 0$.*