Mathematical Analysis 1:
Tutorial #9

Irena Penev
Summer 2026

Exercise 3 of Tutorial 8. *Let $a > 0$ be a fixed constant. Prove that $\left|\ln\left(\frac{x_2}{x_1}\right)\right| < \frac{|x_2 - x_1|}{a}$ for all distinct $x_1, x_2 \in (a, +\infty)$.*

Exercise 6 of Tutorial 8. *Let $a, b \in \mathbb{R}$ be such that $a < b$, and let $f, g, h : [a, b] \to \mathbb{R}$ be functions that are continuous on $[a, b]$ and differentiable on (a, b). Prove that there exists a point $c \in (a, b)$ such that*

$$\begin{vmatrix} f'(c) & g'(c) & h'(c) \\ f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \end{vmatrix} = 0.$$

Hint: *Construct a suitable function $F : [a, b] \to \mathbb{R}$ given by*

$$F(x) := \begin{vmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{vmatrix} \qquad \forall x \in [a, b].$$

Exercise 1. *Find two real numbers whose difference is 100 and whose product is minimum.*

Exercise 2. *Find two positive real numbers whose product is 100 and whose sum is minimum.*

Exercise 3. *Find the dimensions of a rectangle with perimeter $100m$ whose area is as large as possible.*

Exercise 4. *A box with a square base and open top must have a volume of $32000 cm^3$. Find the dimensions of the box that minimize the amount of material used.*

Exercise 5. *Find the dimensions of the rectangle of largest area that can be inscribed in an equilateral triangle of side L if one side of the rectangle lies on the base of the triangle.*

Exercise 6. *Sketch the graphs of the following functions:*

(a) $f(x) = \frac{\ln x}{x^2}$; *(b)* $f(x) = x - 3\sqrt[3]{x}$; *(c)* $f(x) = xe^{-x^2}$.