

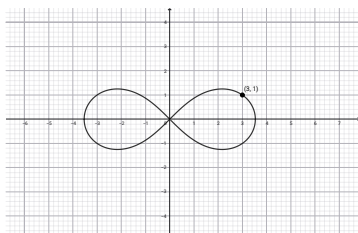
Mathematical Analysis 1:

Tutorial #7 - some solutions

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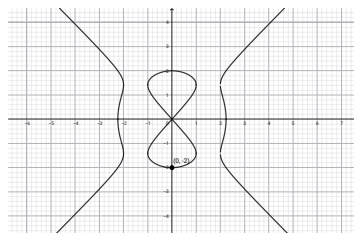
Exercise 5. Use implicit differentiation to compute the equation of the tangent to the given curve through the specified point.

(a) $2(x^2 + y^2)^2 = 25(x^2 - y^2)$, $(3, 1)$;



(lemniscate)

(b) $y^2(y^2 - 4) = x^2(x^2 - 5)$, $(0, -2)$.



(devil's curve)

Solution. (a) We differentiate both sides and solve for $\frac{dy}{dx}$, as follows:

$$\begin{aligned} \frac{d}{dx} \left(2(x^2 + y^2)^2 \right) &= \frac{d}{dx} \left(25(x^2 - y^2) \right) \\ 2 \cdot 2(x^2 + y^2)(2x + 2y \frac{dy}{dx}) &= 25(2x - 2y \frac{dy}{dx}) \\ 2(x^2 + y^2)(2x + 2y \frac{dy}{dx}) &= 25(x - y \frac{dy}{dx}) \\ 4x(x^2 + y^2) + 4y(x^2 + y^2) \frac{dy}{dx} &= 25x - 25y \frac{dy}{dx} \\ y(4x^2 + 4y^2 + 25) \frac{dy}{dx} &= x(-4x^2 - 4y^2 + 25) \\ \frac{dy}{dx} &= \frac{x}{y} \cdot \frac{-4x^2 - 4y^2 + 25}{4x^2 + 4y^2 + 25} \end{aligned}$$

This implies that

$$\left. \frac{dy}{dx} \right|_{(x,y)=(3,1)} = \frac{3}{1} \cdot \frac{-4 \cdot 3^2 - 4 \cdot 1^2 + 25}{4 \cdot 3^2 + 4 \cdot 1^2 + 25} = 3 \cdot \frac{-15}{65} = -\frac{9}{13},$$

and so the equation of the tangent to our curve through the point $(3, 1)$ is

$$y = -\frac{9}{13}(x - 3) + 1,$$

that is,

$$y = -\frac{9}{13}x + \frac{40}{13}.$$

(b) We differentiate both sides and solve for $\frac{dy}{dx}$, as follows:

$$\frac{d}{dx}(y^2(y^2 - 4)) = \frac{d}{dx}(x^2(x^2 - 5))$$

$$\frac{d}{dx}(y^4 - 4y^2) = \frac{d}{dx}(x^4 - 5x^2)$$

$$4y^3 \frac{dy}{dx} - 8y \frac{dy}{dx} = 4x^3 - 10x$$

$$2y^3 \frac{dy}{dx} - 4y \frac{dy}{dx} = 2x^3 - 5x$$

$$(2y^3 - 4y) \frac{dy}{dx} = 2x^3 - 5x$$

$$\frac{dy}{dx} = \frac{2x^3 - 5x}{2y^3 - 4y}$$

This implies that

$$\left. \frac{dy}{dx} \right|_{(x,y)=(0,-2)} = \frac{2 \cdot 0^3 - 5 \cdot 0}{2 \cdot (-2)^3 - 4(-2)} = 0,$$

and so the equation of the tangent to our curve through the point $(0, -2)$ is

$$y = 0(x - 0) + (-2),$$

that is,

$$y = -2.$$

□