Mathematical Analysis 1:
Tutorial #1 (Exercise 1(t) - solution)

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Exercise 1. *Either evaluate the following limits, or explain why they do not exist. (If the sequence diverges to $+\infty$ or $-\infty$, then this should be indicated.)*

(t) $\lim\limits_{n\to\infty} \big(\sqrt[3]{n^3+2n^2} - \sqrt[3]{n^3-n}\big)$

Solution. We recall the formula $(x-y)(x^2+xy+y^2) = x^3-y^3$. We now compute the limit as follows:

$$\begin{aligned}
\lim_{n\to\infty} \big(\sqrt[3]{n^3+2n^2} - \sqrt[3]{n^3-n}\big) &= \lim_{n\to\infty} \frac{\big(\sqrt[3]{n^3+2n^2} - \sqrt[3]{n^3-n}\big)\Big((\sqrt[3]{n^3+2n^2})^2 + \sqrt[3]{n^3+2n^2}\,\sqrt[3]{n^3-n} + (\sqrt[3]{n^3-n})^2\Big)}{(\sqrt[3]{n^3+2n^2})^2 + \sqrt[3]{n^3+2n^2}\,\sqrt[3]{n^3-n} + (\sqrt[3]{n^3-n})^2} \\
&= \lim_{n\to\infty} \frac{(n^3+2n^2)-(n^3-n)}{\sqrt[3]{n^6+4n^5+4n^4} + \sqrt[3]{n^6+2n^5-n^4-2n^3} + \sqrt[3]{n^6-2n^4+n^2}} \\
&= \lim_{n\to\infty} \frac{2n^2+n}{n^2\Big(\sqrt[3]{1+\frac{4}{n}+\frac{4}{n^2}} + \sqrt[3]{1+\frac{2}{n}-\frac{1}{n^2}-\frac{2}{n^3}} + \sqrt[3]{1-\frac{2}{n^2}+\frac{1}{n^4}}\Big)} \\
&= \lim_{n\to\infty} \frac{2+\frac{1}{n}}{\sqrt[3]{1+\frac{4}{n}+\frac{4}{n^2}} + \sqrt[3]{1+\frac{2}{n}-\frac{1}{n^2}-\frac{2}{n^3}} + \sqrt[3]{1-\frac{2}{n^2}+\frac{1}{n^4}}} \\
&\overset{(*)}{=} \frac{2}{3},
\end{aligned}$$

where for (*), we used the fact that for all fixed $k \in \mathbb{N}$, we have that $\lim\limits_{n\to\infty} \frac{1}{n^k} = 0$. □