Mathematical Analysis 1: Tutorial #1

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Exercise 1. *Either evaluate the following limits, or explain why they do not exist. (If the sequence diverges to $+\infty$ or $-\infty$, then this should be indicated.)*

(a) $\lim\limits_{n\to\infty} \frac{(n+1)^3-(n-1)^2}{(n+1)^3+(n-1)^3}$

(b) $\lim\limits_{n\to\infty} \frac{(2n+1)^4-(n-1)^4}{(2n+1)^4+(n-1)^4}$

(c) $\lim\limits_{n\to\infty} \left(\frac{2n^2}{2n+3} + \frac{1-3n^3}{3n^2+1}\right)$

(d) $\lim\limits_{n\to\infty} \left(\frac{3n^2}{2n+1} + \frac{1-6n^3}{1+4n^2}\right)$

(e) $\lim\limits_{n\to\infty} \frac{3n^3+5n-4}{4n^3-5n^2+1}$

(f) $\lim\limits_{n\to\infty} \frac{3n^3+5n-4}{7n^5-3n^3+1}$

(g) $\lim\limits_{n\to\infty} \frac{3n^5+5n^2-4}{3n^4-2n^2-7}$

(h) $\lim\limits_{n\to\infty} \frac{n!}{(n+1)!-n!}$

(i) $\lim\limits_{n\to\infty} \frac{n!}{2^n}$

(j) $\lim\limits_{n\to\infty} \frac{2^n-1}{2^n+1}$

(k) $\lim\limits_{n\to\infty} \frac{2^{\frac{1}{n}}-1}{2^{\frac{1}{n}}+1}$

(l) $\lim\limits_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n}$

(m) $\lim\limits_{n\to\infty} \frac{\sum_{i=0}^{n} a^i}{\sum_{i=0}^{n} b^i}$, *where* $a, b \in \mathbb{R}$, *with* $|a|, |b| < 1$

(n) $\lim\limits_{n\to\infty} \frac{\sin n}{n}$

(o) $\lim\limits_{n\to\infty} \frac{\cos^2 n}{2^n}$

(p) $\lim\limits_{n\to\infty} \frac{3n^3+\cos(n^{27})}{5n^3+3n}$

(q) $\lim\limits_{n\to\infty} \left(\sqrt{2n^2+7n-3} - \sqrt{2n^2+3n-7}\right)$

(r) $\lim\limits_{n\to\infty} \left(\sqrt{n^2+3n} - \sqrt{n^2+n-1}\right)$

(s) $\lim\limits_{n\to\infty} \left(\sqrt{n} - \sqrt{n^2-1}\right)$

(t) $\lim\limits_{n\to\infty} \left(\sqrt[3]{n^3+2n^2} - \sqrt[3]{n^3-n}\right)$

Exercise 2.

(a) *Either give an example of divergent sequences* $\{a_n\}_{n=1}^{\infty}$ *and* $\{b_n\}_{n=1}^{\infty}$ *such that* $\{a_n+b_n\}_{n=1}^{\infty}$ *is convergent, or explain why such sequences do not exist.*

(b) *Either give an example of divergent sequences* $\{a_n\}_{n=1}^{\infty}$ *and* $\{b_n\}_{n=1}^{\infty}$ *such that* $\{a_nb_n\}_{n=1}^{\infty}$ *is convergent, or explain why such sequences do not exist.*

(c) *Either give an example of a convergent sequence* $\{a_n\}_{n=1}^{\infty}$ *and a divergent sequence* $\{b_n\}_{n=1}^{\infty}$ *such that* $\{a_n+b_n\}_{n=1}^{\infty}$ *is convergent, or explain why such sequences do not exist.*

(d) *Either give an example of a convergent sequence* $\{a_n\}_{n=1}^{\infty}$ *and a divergent sequence* $\{b_n\}_{n=1}^{\infty}$ *such that* $\{a_nb_n\}_{n=1}^{\infty}$ *is convergent, or explain why such sequences do not exist.*