Mathematical Analysis 1:
Practice problems #5 (not to be turned in)

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There will be a quiz on Thursday, April 30. The quiz will consist of two problems, as follows.

1. In the first problem, you will be asked to compute a few derivatives, similar to those from Problem 1 below.

2. In the second problem, you will be asked to compute a few limits, similar to those from Exercise 1 of Tutorial 8 (see `https://iuuk.mff.cuni.cz/~ipenev/MA1S2026Tutorial08`). In some (but probably not all) of the limits, L'Hôpital's Rule will apply. Don't forget that you can only apply L'Hôpital's Rule to indeterminate forms of type "$\frac{0}{0}$" and "$\frac{\infty}{\infty}$"!

Problems 2, 3, and 4 (below) will **not** be on the quiz, but similar problems may be on the exam. You may study any way you like, but on the quiz itself, you will **not** be able to use any pre-prepared notes or electronic devices. However, I suggest that you prepare by trying to solve the problems by yourself first, and if you aren't able to do so, then get help from someone (such as myself or a classmate) or something (such as AI, though keep in mind that AI occasionally makes mistakes).

Problem 1. *Compute the derivatives of the following:*

1. x^7;
2. 7^x;
3. $7^{\sin x}$;
4. $\sqrt[3]{x} + \frac{1}{\sqrt[3]{x}}$;
5. $(4x)^{3x}$;
6. $\frac{x}{1+2x^2}$;
7. $\frac{1}{\sqrt{1+5x^3}}$;
8. $\frac{\ln 4}{x^5}$;
9. $\frac{\cos x}{e^5}$;
10. $(\ln x)^{\sin x}$;
11. $\sin(\pi x)$;
12. $\cos(\frac{\pi}{2}x)$;
13. $\sin^7(x)$;
14. $\sin(x^7)$;
15. $\sin^7(x^7)$;
16. $\sin^2(x^3)$;
17. $\log_x 13$;
18. $\log_{2x}(3x)$;
19. $\log_7(\frac{1}{x})$;
20. $(\log_2 3)(\ln x)x$;
21. $\arctan(\sqrt{x})$;
22. $\sqrt{\arctan x}$;
23. $\arctan(\arcsin x)$;
24. $\arcsin(\arctan x)$;
25. $(\arcsin x)(\arctan x)$;
26. $\arcsin(e^{2x})$;
27. $\ln(\arcsin x)$;
28. $(\tan x)^{\cos x}$;
29. $(\cos x)^{\ln x}$;
30. $(7x)^{\sqrt{x}}$;
31. $\frac{e^x}{\sqrt{x}}$;
32. $\frac{e}{x^5}$;
33. x^e;
34. $\cos(\sin x)$;
35. $\cos^3(\tan x)$;
36. $\cos^3(e^x \ln x)$.

Problem 2. *Using Lagrange's Mean Value Theorem, prove that for all $x_1, x_2 \in \mathbb{R}$, we have that $|\sin x_1 - \sin x_2| \leq |x_1 - x_2|$.*

Problem 3. *Using Lagrange's Mean Value Theorem, prove that for all $x_1, x_2 \in [0, +\infty)$, we have that $|e^{x_1} - e^{x_2}| \geq |x_1 - x_2|$.*

Problem 4. *Let $C : \mathbb{R} \setminus \{-1, 1\} \to \mathbb{R}$ be given by*

$$C(x) \quad := \quad \begin{cases} \frac{\pi}{2} & \text{if} \quad x < -1 \\ 0 & \text{if} \quad -1 < x < 1 \\ -\frac{\pi}{2} & \text{if} \quad x > 1 \end{cases}$$

for all $x \in \mathbb{R} \setminus \{-1, 1\}$. Using derivatives, prove that

$$\tfrac{1}{2} \arctan\left(\tfrac{2x}{1-x^2}\right) \quad = \quad \arctan x + C(x)$$

for all $x \in \mathbb{R} \setminus \{-1, 1\}$.

> ***Hint:*** *This is similar to Exercise 2 from Tutorial 8 (`https://iuuk.mff.cuni.cz/~ipenev/MA1S2026Tutorial08Ex2-soln`), but a bit more complicated. Also, points $x = -\sqrt{3}$, $x = 0$, and $x = \sqrt{3}$ are useful to consider.*