Mathematical Analysis 1: Practice problems #3 (not to be turned in)

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One of the first three problems will be on the quiz on Thursday, March 26. (Different students will get different problems.) Problem 4 will **not** be on the quiz, but similar problems may be on the exam. You may study any way you like, but on the quiz itself, you will need to solve these problems **without** using any pre-prepared notes or electronic devices. However, I suggest that you prepare by trying to solve the problems by yourself first, and if you aren't able to do so, then get help from someone (such as myself or a classmate) or something (such as AI, though keep in mind that AI occasionally makes mistakes). The definition of the limit of a function is given below, for your convenience. However, on the quiz, you will **not** be given the definition, and so make sure you have it memorized.

Definition. *An* accumulation point *of a set* $A \subseteq \mathbb{R}$ *is a point* $a \in \mathbb{R}$ *(note that* a *may or may not belong to* A*) such that for all real numbers* $\varepsilon > 0$*, there exists some* $a' \in A$ *such that* $0 < |a' - a| < \varepsilon$*.*

Definition (The ε-δ definition of a limit)**.** *Let* $f : A \subseteq \mathbb{R} \to \mathbb{R}$ *be a function, let* $a \in \mathbb{R}$ *be an accumulation point of* A*, and let* $L \in \mathbb{R}$*. We say that* L is the limit of $f(x)$ as x approaches a*, or that* $f(x)$ tends to L as x approaches a*, provided that the following holds:*

> *for every* $\varepsilon > 0$*, there exists some* $\delta > 0$*, such that for all* $x \in A$*, if* $0 < |x - a| < \delta$*, then* $|f(x) - L| < \varepsilon$*.*

Under such circumstances, we write

$$L = \lim_{x \to a} f(x) \qquad \text{or} \qquad f(x) \to L \quad \text{as} \quad x \to a.$$

Problem 1. *Consider the function* $f : \mathbb{R} \setminus \{5\} \to \mathbb{R}$ *given by*

$$f(x) \quad := \quad \tfrac{x^2 - 2x - 15}{x - 5}$$

for all $x \in \mathbb{R} \setminus \{5\}$*. Using the* ***ε-δ definition****, prove that* $\lim_{x \to 5} f(x) = 8$*.*

Problem 2. *Consider the function* $f : \mathbb{R} \to \mathbb{R}$ *given by*

$$f(x) \quad := \quad \begin{cases} (x-1)^2 & \textit{if} \quad x \neq 1 \\ 100 & \textit{if} \quad x = 1 \end{cases}$$

for all $x \in \mathbb{R}$*. Using the* ***ε-δ definition****, prove that* $\lim_{x \to 1} f(x) = 0$*.*

Problem 3. *Consider the function* $f : \mathbb{R} \to \mathbb{R}$ *given by*

$$f(x) \quad := \quad \begin{cases} x & \textit{if } x \textit{ is rational} \\ 0 & \textit{if } x \textit{ is irrational} \end{cases}$$

for all $x \in \mathbb{R}$*. Using the* ***ε-δ definition****, prove that* $\lim_{x \to 0} f(x) = 0$*.*

Problem 4. *Consider the function* $f : \mathbb{R} \to \mathbb{R}$ *given by*

$$f(x) \quad := \quad \begin{cases} x-2 & \textit{if} \quad x > 1 \\ -5 & \textit{if} \quad x = 1 \\ -x^3 & \textit{if} \quad x < 1 \end{cases}$$

for all $x \in \mathbb{R}$*. Using the* ***ε-δ definition****, prove that* $\lim_{x \to 1} f(x) = -1$*.*