Mathematical Analysis 1: Practice problems #1 (not to be turned in)

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Two of the the following six problems will be on the quiz on Thursday, February 26, 2026: you will get one of the first three problems, and one of the last three. (Different students will get different problem combinations.) You may study any way you like, but on the quiz itself, you will need to solve these problems **without** using any pre-prepared notes or electronic devices. However, I suggest that you prepare by trying to solve the problems by yourself first, and if you aren't able to do so, then get help from someone (such as myself or a classmate) or something (such as AI, though keep in mind that AI occasionally makes mistakes). The definition of convergence/divergence and of a limit is given below, for your convenience. However, on the quiz, you will **not** be given the definition, and so make sure you have it memorized.

Definition 1. *We say that a sequence* $\{a_n\}_{n=1}^{\infty}$ *of real numbers* converges *to a real number* L *provided that the following holds:*

> *For all real numbers* $\varepsilon > 0$*, there exists some* $N \in \mathbb{N}$ *such that for all* $n \in \mathbb{N}$*: if* $n \geq N$*, then* $|a_n - L| < \varepsilon$*.*

Under such circumstances, we say that L *is the* limit *of the sequence* $\{a_n\}_{n=1}^{\infty}$*, and we write*

$$L = \lim_{n \to \infty} a_n$$

or

$$a_n \to L \quad \text{as} \quad n \to \infty.$$

A sequence is convergent *(or* converges*) if it has a limit. Otherwise, it is* divergent *(or* diverges*).*

Problem 1. *Using the **definition** (with* ε*), prove that* $\lim\limits_{n \to \infty} \left(\frac{(-1)^n}{n} \right) = 0$.

Problem 2. *Using the **definition** (with* ε*), prove that* $\lim\limits_{n \to \infty} \big(7 + \frac{1}{\sqrt{n}} \big) = 7$.

Problem 3. *Using the **definition** (with* ε*), prove that* $\lim\limits_{n \to \infty} \big(-1 + \frac{1}{2n^2} \big) = -1$.

Problem 4. *Using the* ***definition*** *(with ε), prove that the sequence* $\left\{(-1)^n + \frac{1}{\sqrt{n}}\right\}_{n=1}^{\infty}$ *diverges.*

Problem 5. *Consider the sequence* $\{a_n\}_{n=1}^{\infty}$ *given by*

$$a_n \quad := \quad \begin{cases} 2 & \text{if} \quad n \text{ is even} \\ -\sqrt{n} & \text{if} \quad n \text{ is odd} \end{cases}$$

for all $n \in \mathbb{N}$. Using the ***definition*** *(with ε), prove that the sequence* $\{a_n\}_{n=1}^{\infty}$ *diverges.*

Problem 6. *Consider the sequence* $\{a_n\}_{n=1}^{\infty}$ *defined by*

$$a_n \quad := \quad \begin{cases} 3 & \text{if} \quad 3 \mid n \\ 7 & \text{if} \quad 3 \nmid n \end{cases}$$

for all $n \in \mathbb{N}$. Using the ***definition*** *(with ε), prove that the sequence* $\{a_n\}_{n=1}^{\infty}$ *diverges.*

> ***Notation:*** *"$m \mid n$" means "n is divisible by m," whereas "$m \nmid n$" means "n is* ***not*** *divisible by m."*