# Linear Algebra 2: Tutorial 11

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**Exercise 6 from Tutorial 10.** *Suppose that $A, B \in \mathbb{R}^{n\times n}$ are symmetric. Show that $AB$ is symmetric if and only if $AB = BA$.*

**Exercise 7 from Tutorial 10.** *Suppose that $A, B \in \mathbb{R}^{n\times n}$ are orthogonally diagonalizable. When is $AB$ orthogonally diagonalizable?*

**Exercise 1.** *Determine whether the following matrices (in $\mathbb{R}^{3\times 3}$) are positive definite:*

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 4 & 2 \\ 0 & 2 & 4 \end{bmatrix}.$$

*For each of the two matrices, do this in three different ways:*

*(a) using row reduction;*

*(b) using determinants;*

*(c) using eigenvalues.*[1]

**Exercise 2.** *Consider the function $\langle \cdot, \cdot \rangle : \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ given by*

*(a)* $\langle \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rangle = 2x_1y_1 + 3x_1y_2 + 3x_2y_1 + 5x_2y_2,$

*(b)* $\langle \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \rangle = 3x_1y_1 + 4x_1y_2 + 4x_2y_1 + 3x_2y_2,$

*for all* $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in \mathbb{R}^2$. *Determine whether $\langle \cdot, \cdot \rangle$ is a scalar product on $\mathbb{R}^2$.*

***Hint:*** *What does this have to do with positive definiteness?*

**Exercise 3.** *Show that any symmetric matrix in $\mathbb{R}^{n\times n}$ is equal to the difference of two positive definite matrices.*

***Hint:*** *Eigenvalues.*

[1] **Hint for factoring:** One eigenvalue of $B$ is 4.